Model AI Assignment: Introduction to Multi-Agent Path Finding

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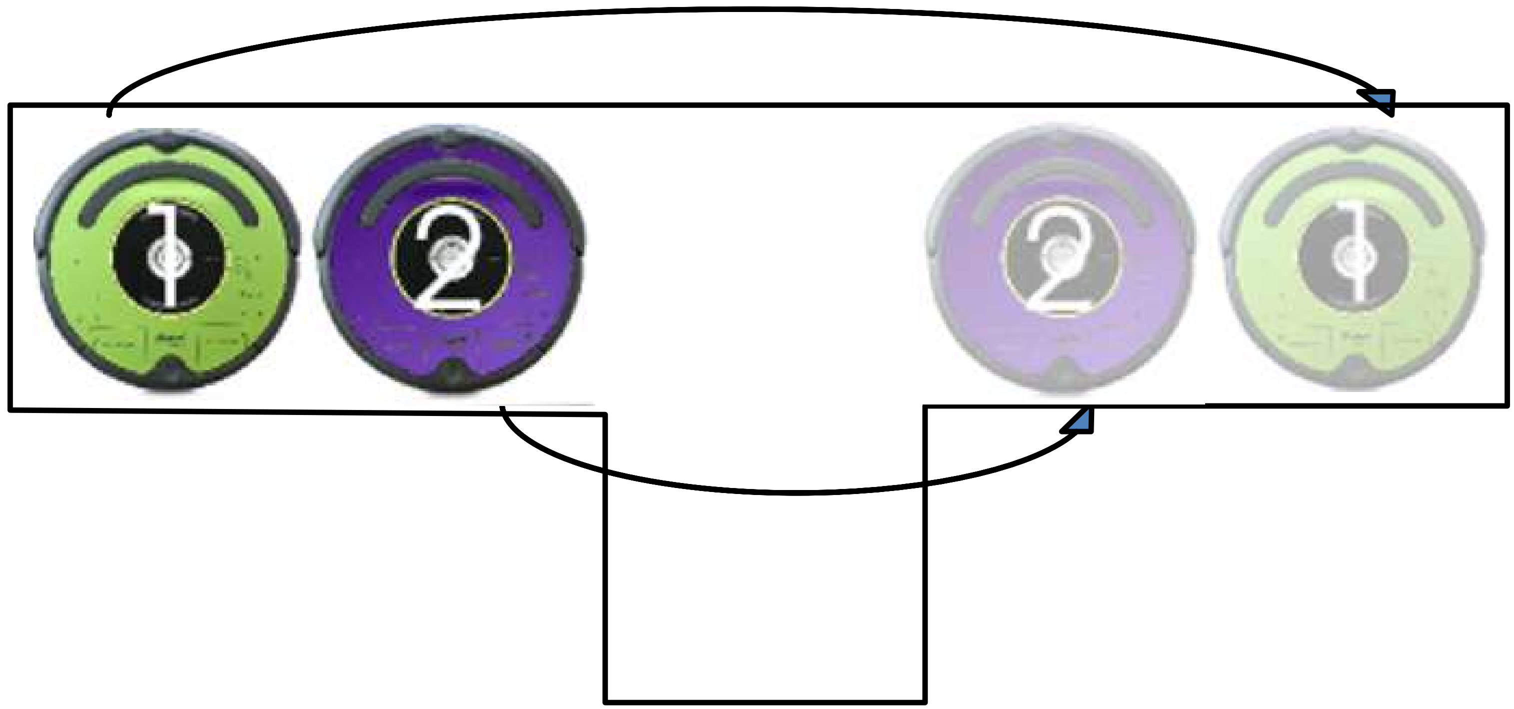
We thank NSF and Amazon Robotics for funding that enabled us to compile this and other teaching material on multi-agent path finding.

Multi-Agent Path Finding (MAPF)



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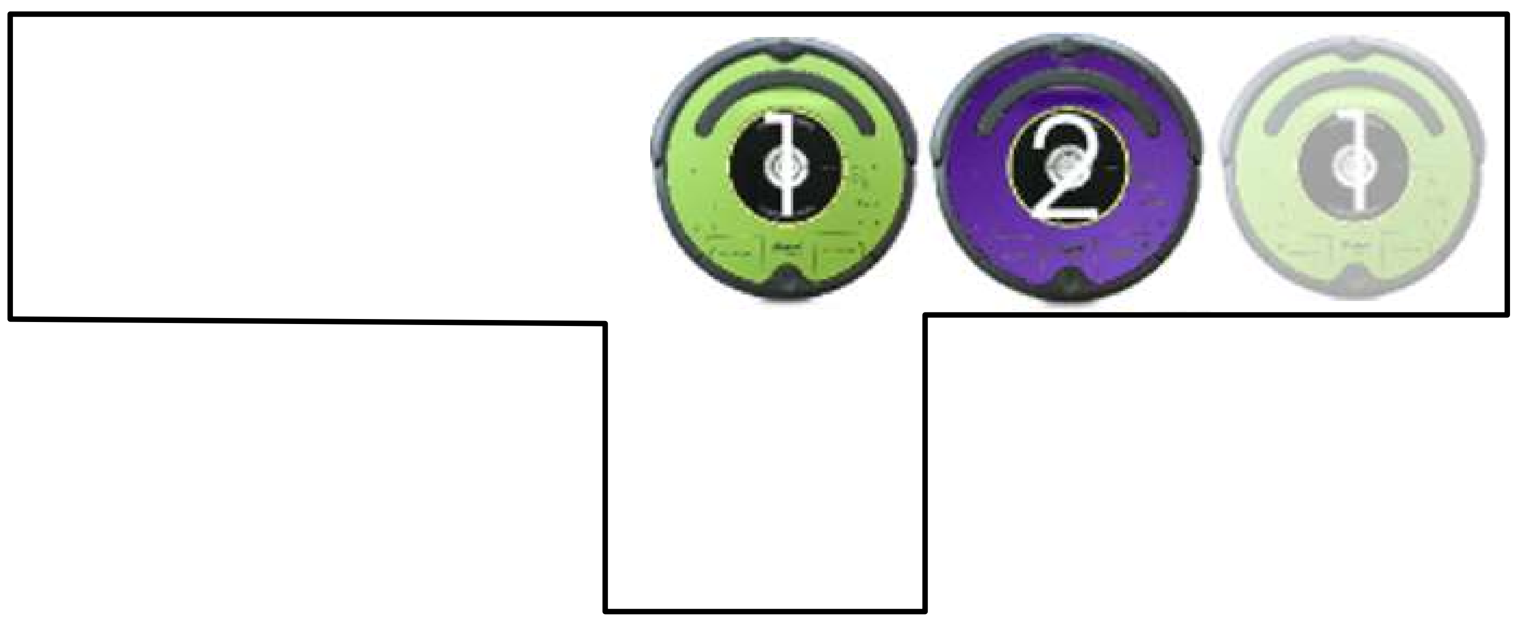

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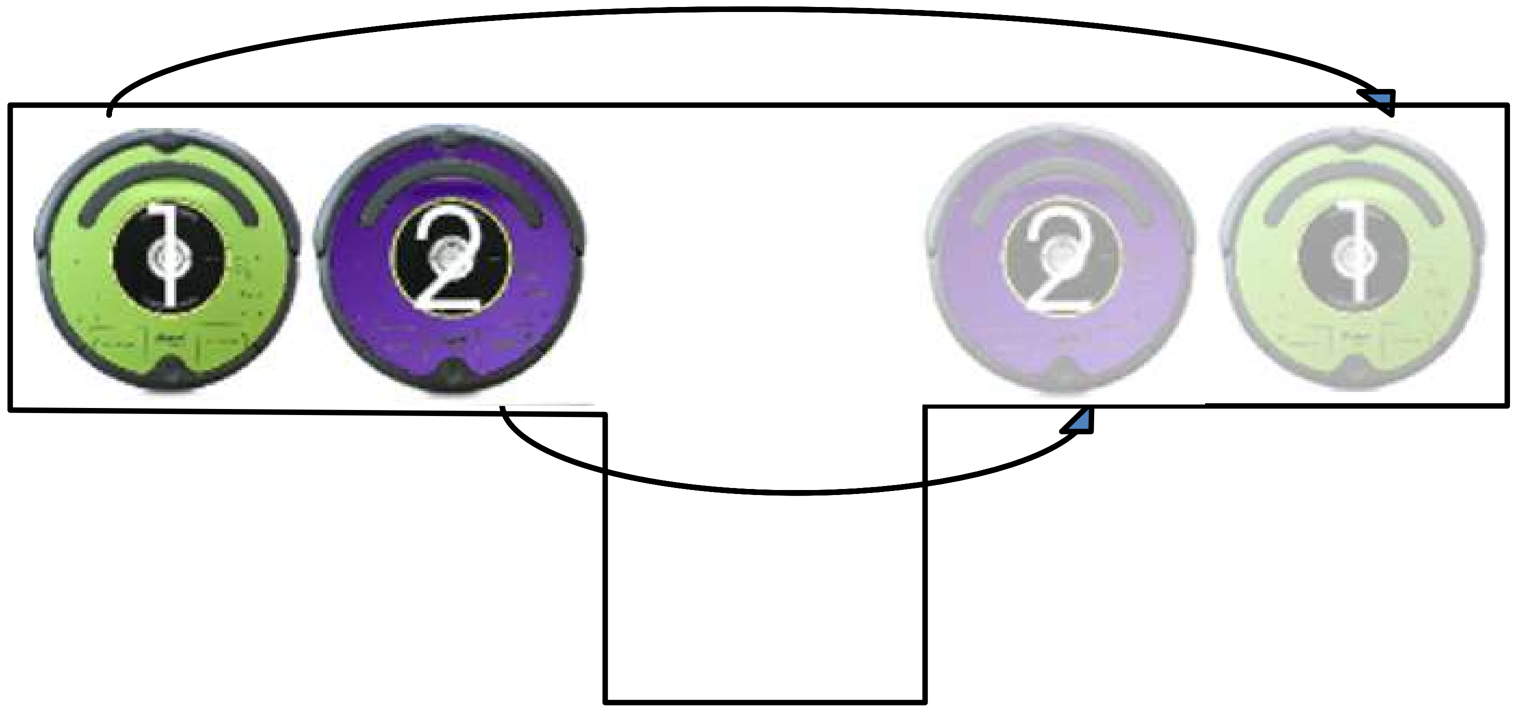

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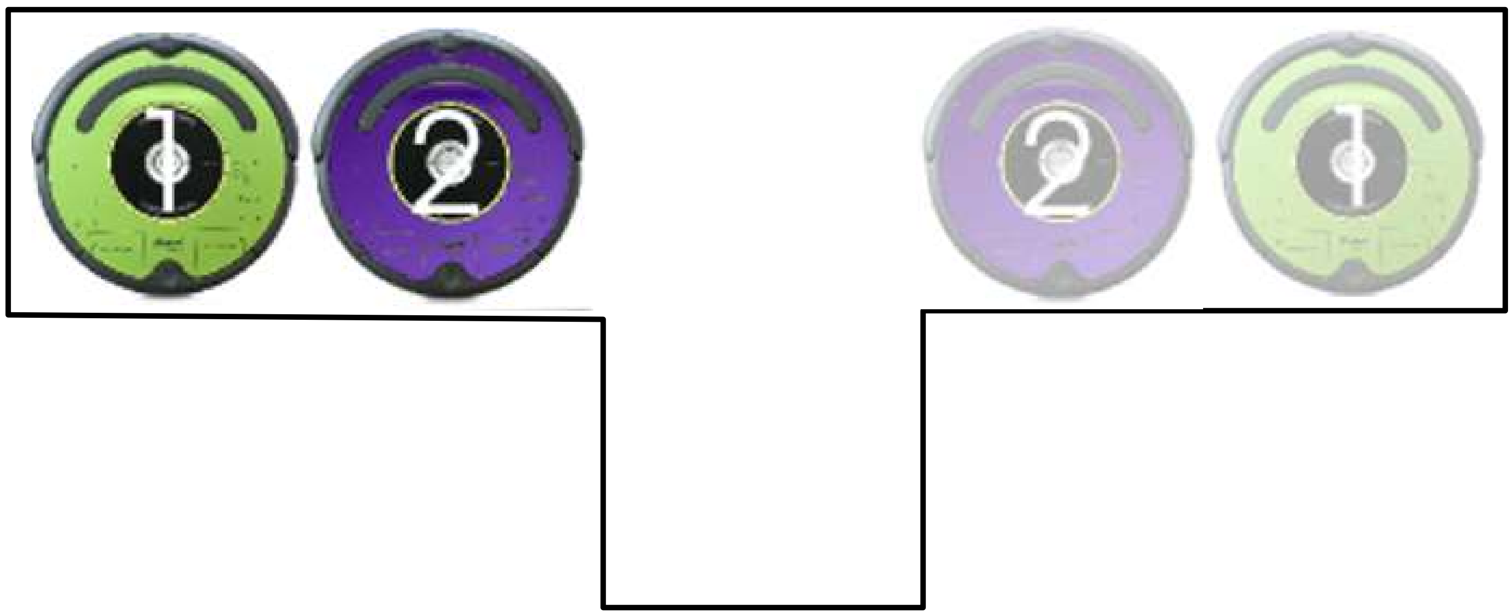

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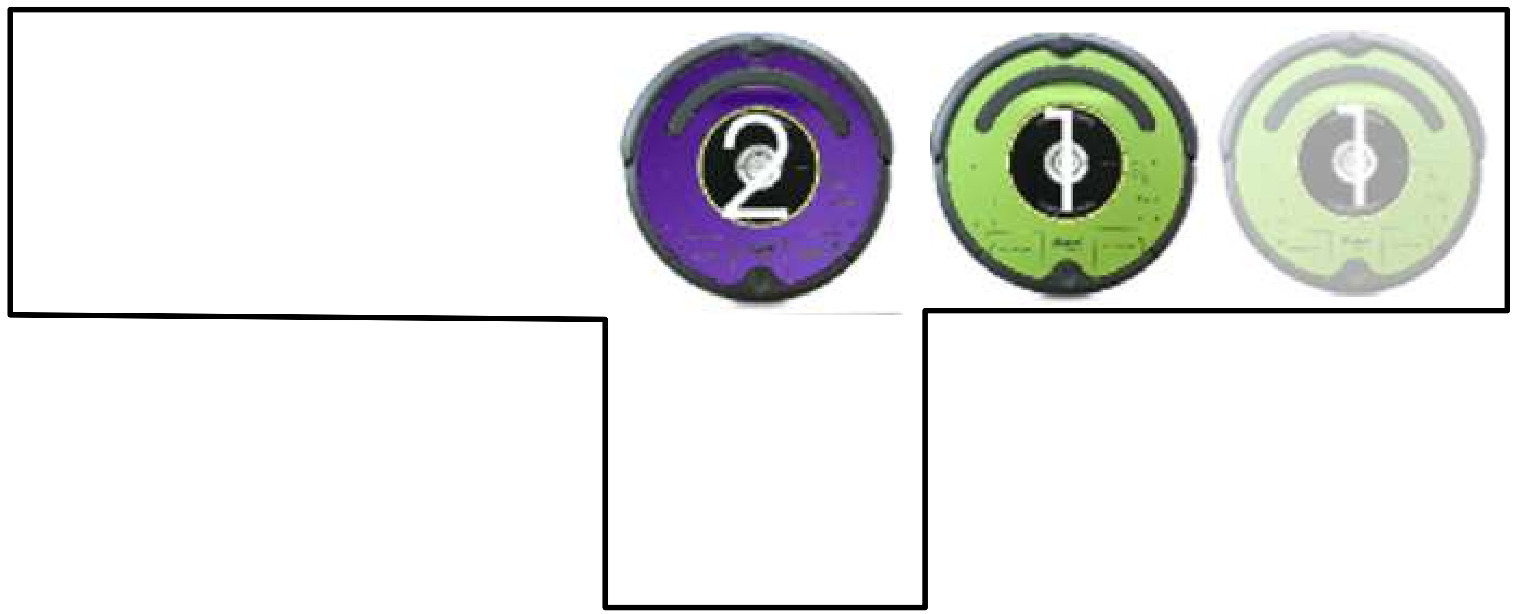
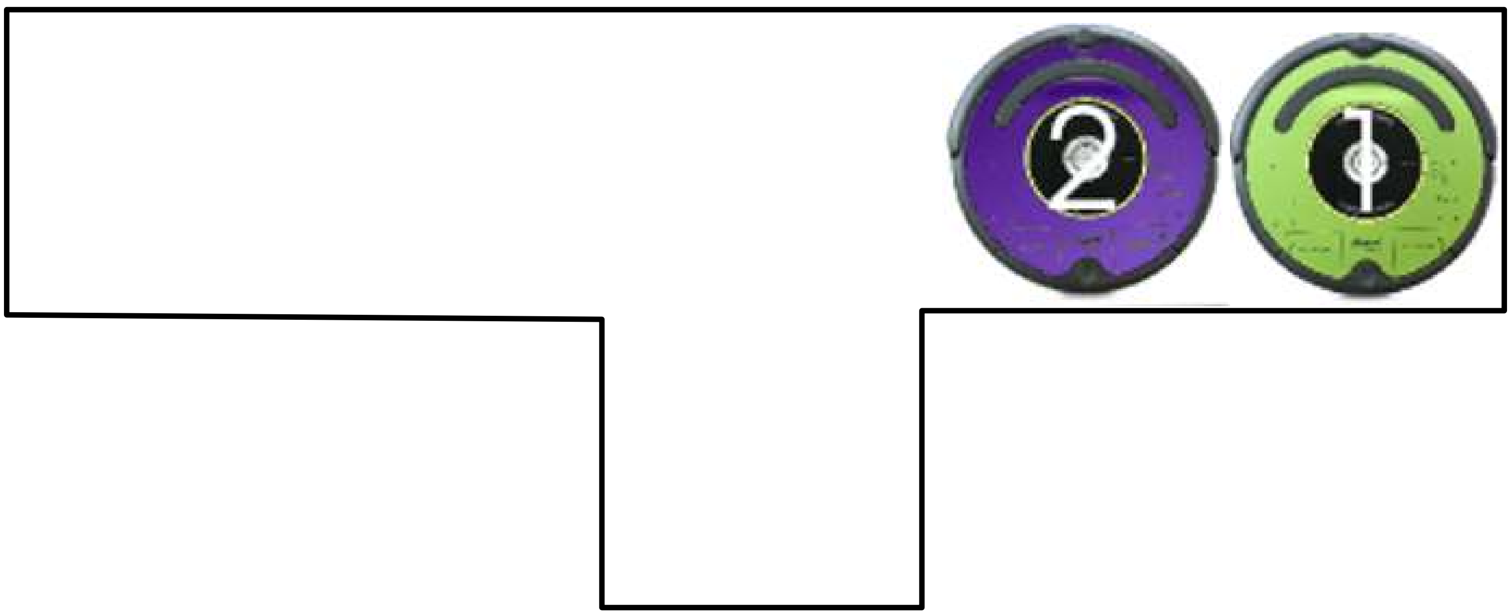


Multi-Agent Path Finding (MAPF)

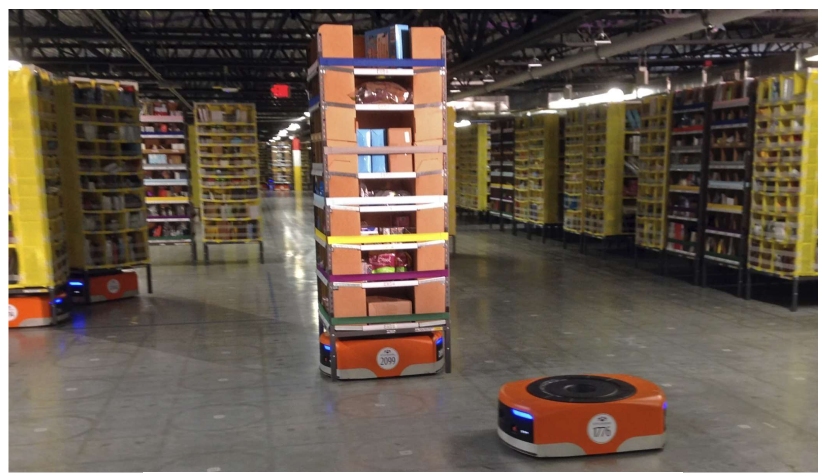


- Optimization problem with the objective
  to minimize task-completion time (called makespan) or
  the sum of travel times (called flowtime)

Multi-Agent Path Finding (MAPF)

- Application: Amazon fulfillment centers
- 2003 Kiva Systems founded
- 2012 Amazon acquires Kiva Systems for $775 million
- 2015 Kiva Systems becomes Amazon Robotics

[www.npr.org – Getty Images]



[www.theguardian.com - AP]

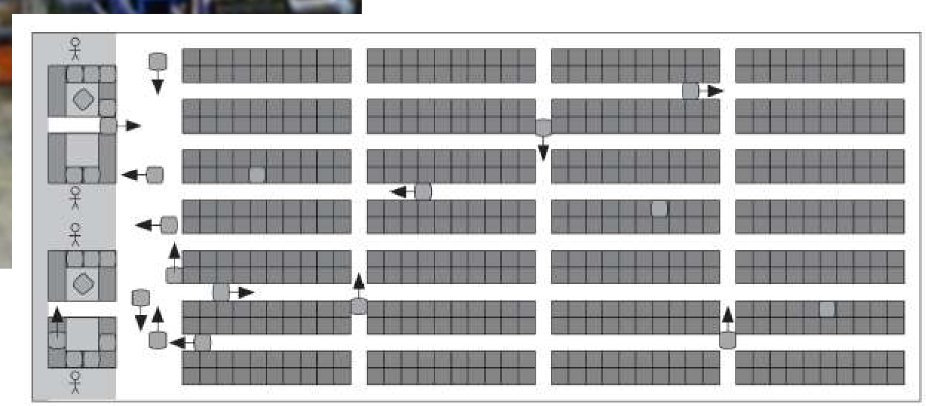
- > 3,000 robots on > 110,000 square meters in Tracy, California

Multi-Agent Path Finding (MAPF)

- Application: Amazon fulfillment centers

[from: YouTube]

amazon



[Wurman, D'Andrea and Mountz]

Multi-Agent Path Finding (MAPF)

- Application: Amazon fulfillment centers

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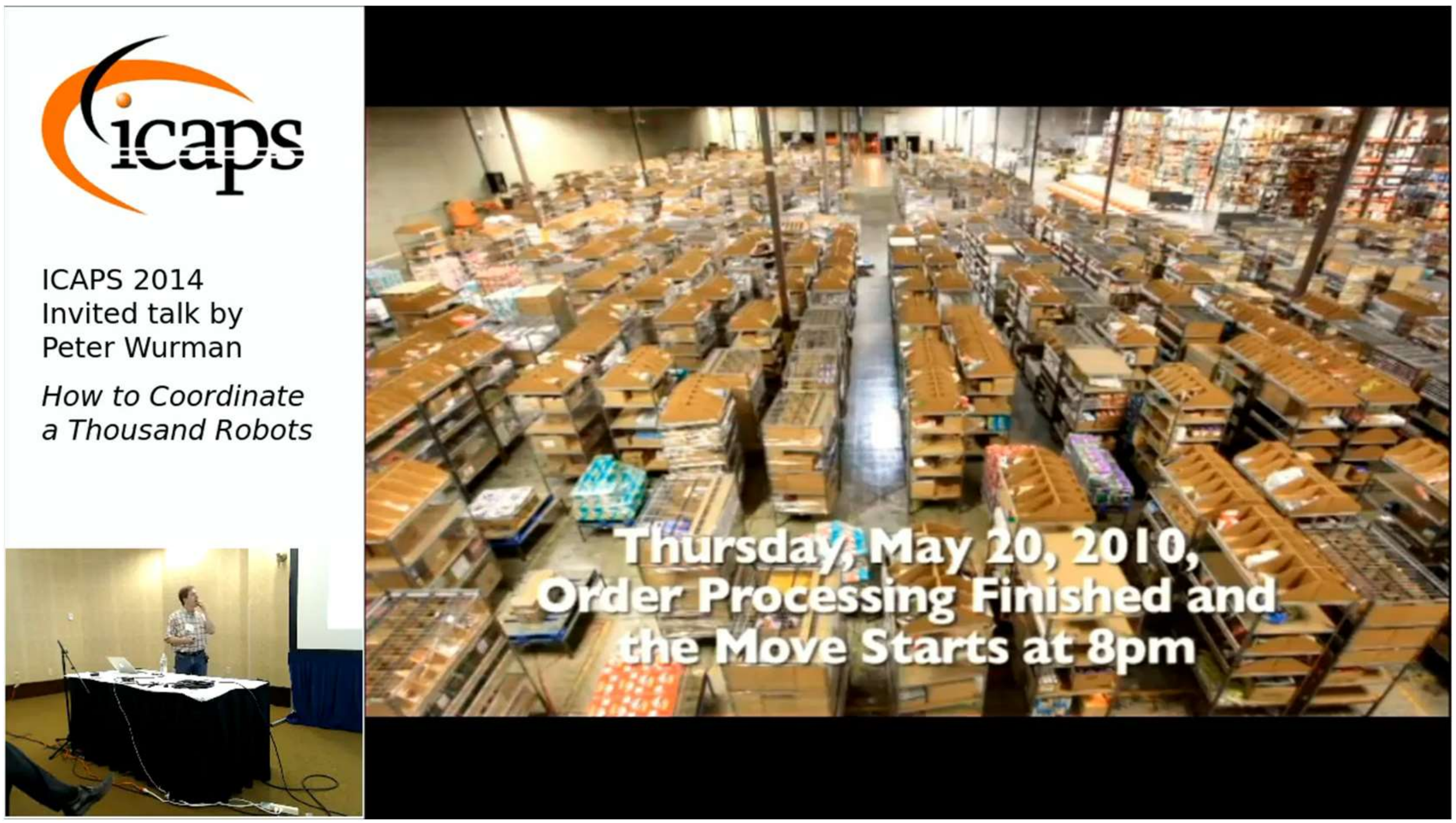
- Application: Amazon fulfillment centers

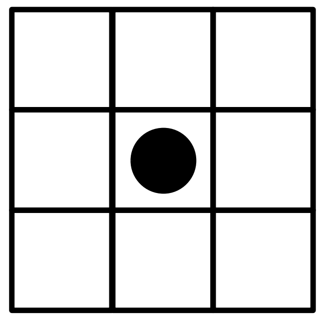


[from: YouTube]

Multi-Agent Path Finding (MAPF)

Robot

Agent



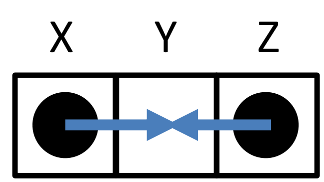
[from: YouTube]

Stickers on the ground establish a grid!

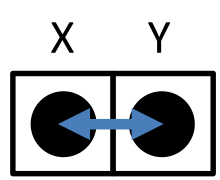
- Simplifying assumptions
  - Point agents
  - No kinematic constraints
  - Discretized environment
    - we use grids here but most techniques work on planar graphs in general

Multi-Agent Path Finding (MAPF)

- Each agent can move N, E, S or W into any adjacent unblocked cell (provided an agent already in that cell leaves it while the agent moves into it or earlier) or wait in its current cell
- Not allowed ("vertex collision")
  - Agent 1 moves from X to Y
  - Agent 2 moves from Z to Y



- Not allowed ("edge collision")
  - Agent 1 moves from X to Y
  - Agent 2 moves from Y to X



Multi-Agent Path Finding (MAPF)

- Suboptimal MAPF algorithms
  – Theorem [Yu and Rus]: MAPF can be solved in polynomial time on undirected grids without makespan or flowtime optimality
  – Unfortunately, good throughput is important in practice!

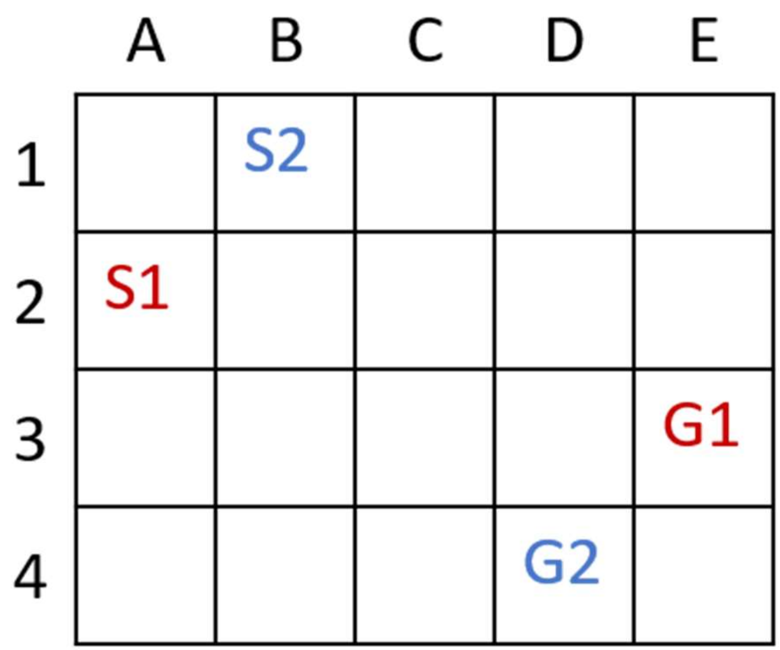
Multi-Agent Path Finding (MAPF)

- Optimal MAPF algorithms
  - Theorem [Yu and LaValle]: MAPF is NP-hard to solve optimally for makespan or flowtime minimization

[www.random-ideas.net]

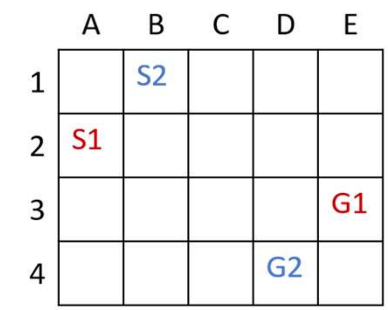
- Bounded-suboptimal MAPF algorithms
  - Theorem [Ma, Tovey, Sharon, Kumar and Koenig]: MAPF is NP-hard to approximate within any factor less than 4/3 for makespan minimization on graphs in general

Multi-Agent Path Finding (MAPF)

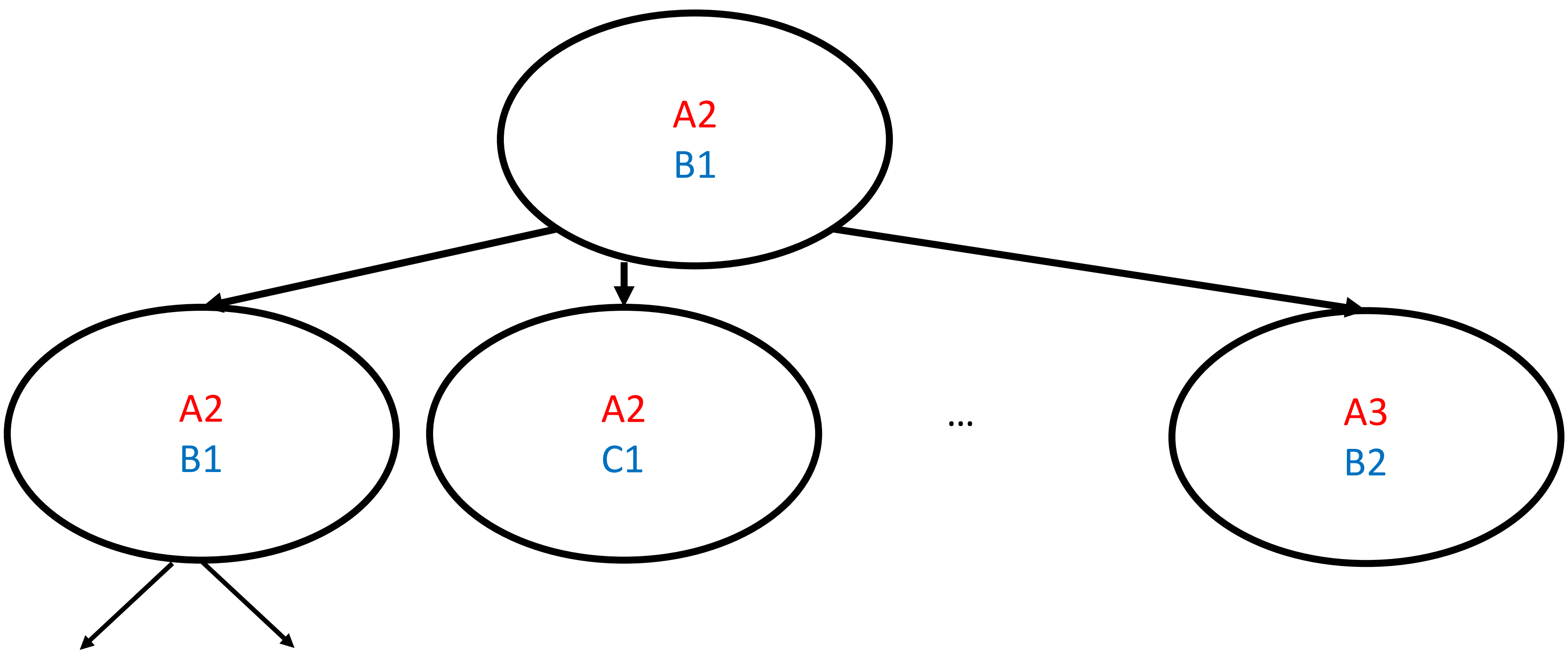


S1 (S2) = start cell of the red (blue) agent
G1 (G2) = goal cell of the red (blue) agent

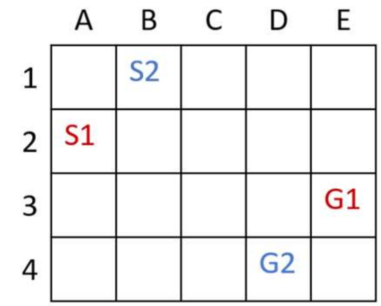
A*-Based Search



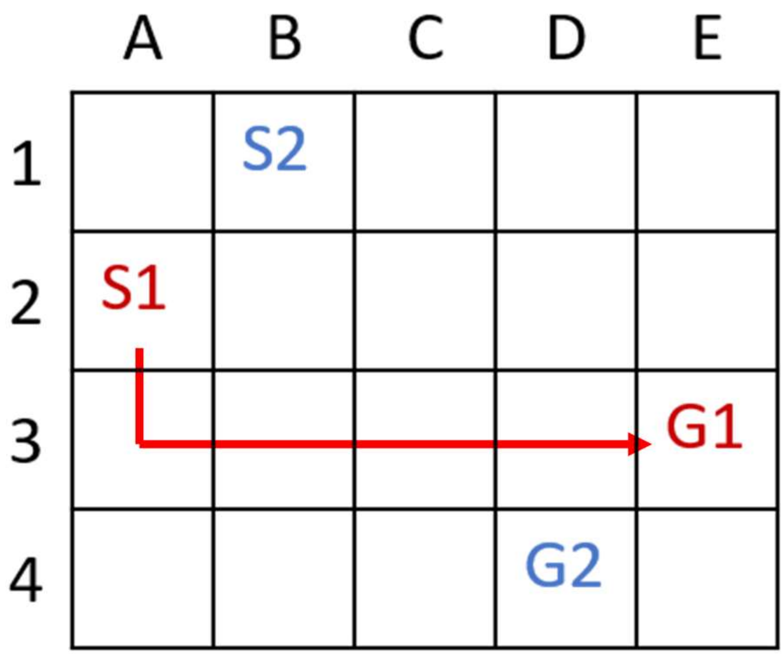
- A*-based search in the joint cell space: Optimal (or bounded-suboptimal) but extremely inefficient MAPF solver



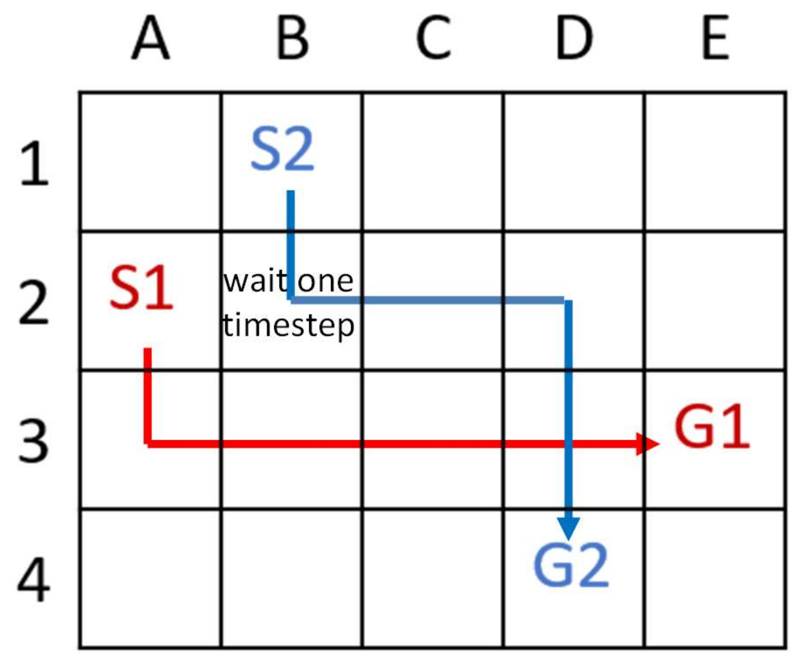
Priority-Based Search



- Priority-based (= sequential) search (plan for one agent after another in space (= cell)-time space in a given order): efficient but suboptimal (and even incomplete) MAPF solver

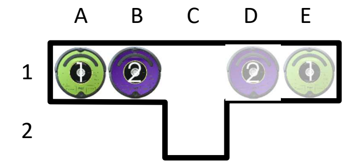


First, find a time-minimal path for the agent with priority 1.



Then, find a time-minimal path for the agent with priority 2 that does not collide with the paths of higher-priority agents.

Priority-Based Search



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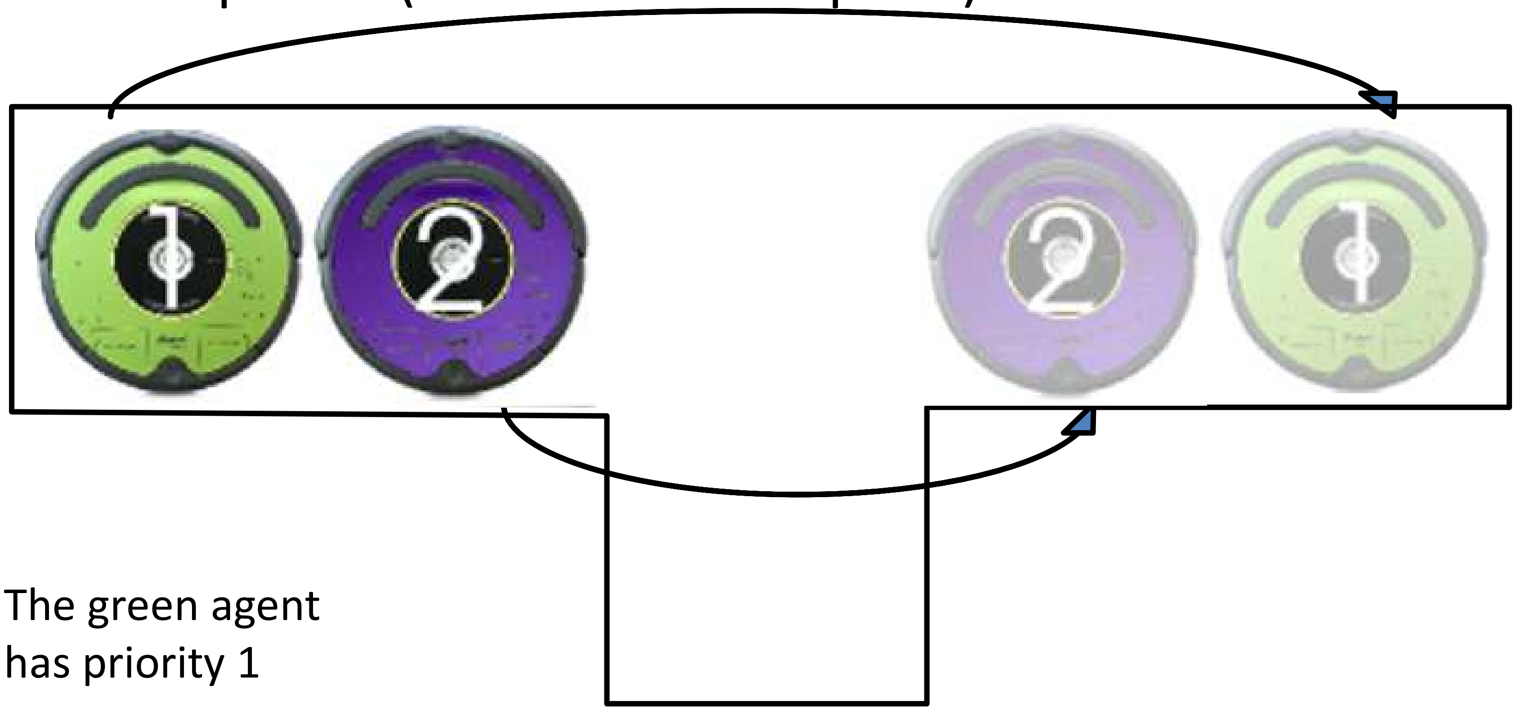
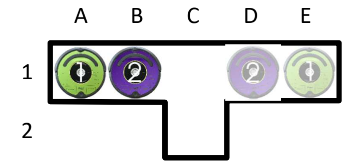


The green agent has priority 1

- Priority-based search finds first path A1, B1, C1, D1, E1 for the green agent and then path B1, C1, C2, C1, D1 for the violet agent. Thus, priority-based search finds a solution.

Priority-Based Search



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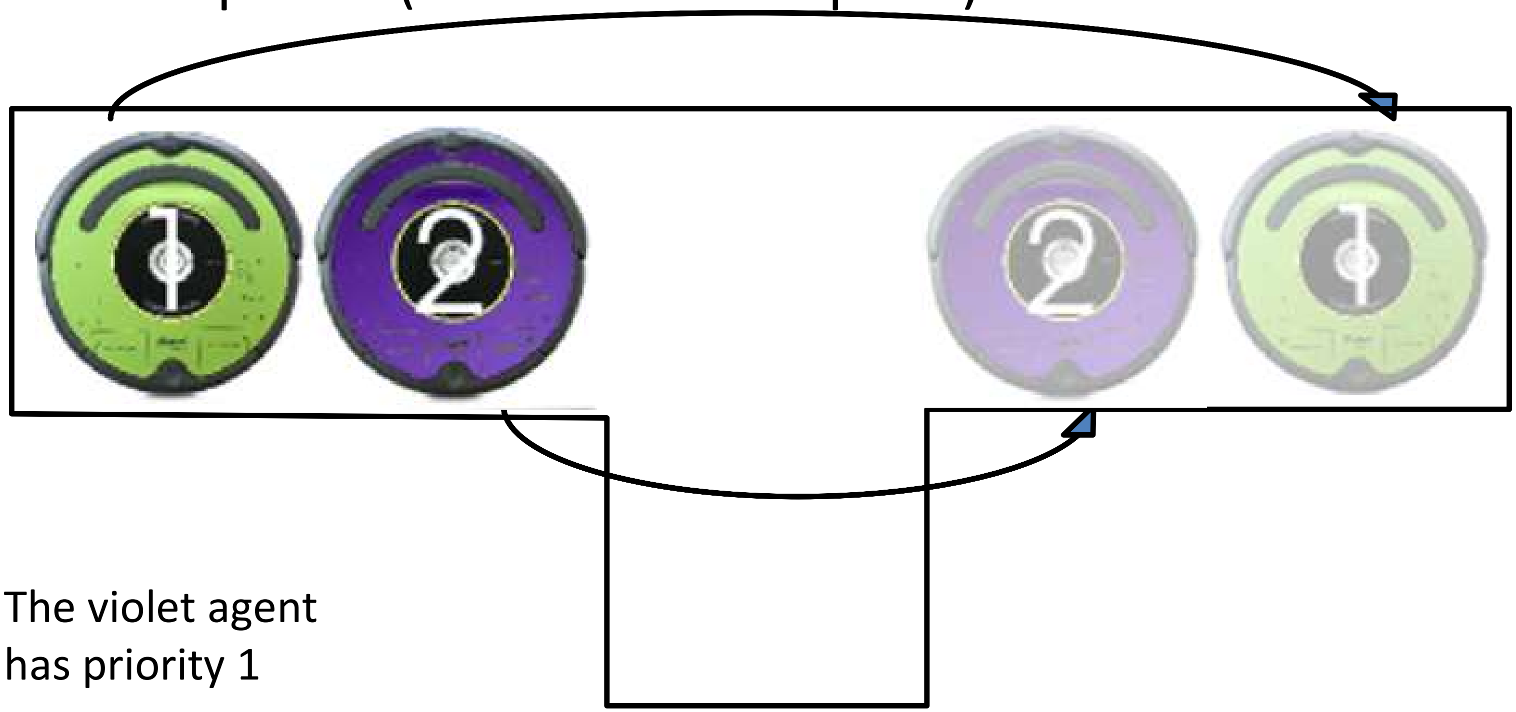


The violet agent has priority 1

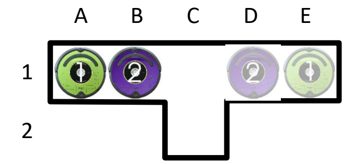
- Priority-based search finds first path B1, C1, D1 for the violet agent and then no path for the green agent. Thus, priority-based search does not find a solution.

Priority-Based Search

- You could implement space (= cell)-time A* with a reservation table (specific for a particular agent) as follows

- The states are pairs (cell, t) for all cells and times
- If the agent can move from cell X to cell Y (in the absence of other agents), create direct edges
  - from state (X,0) to state (Y,1)
  - from state (X,1) to state (Y,2)
  - …
- If the agent is not allowed to be in cell X at time t (because a collision with a higher-priority agent would result), delete state (X,t)
- If the agent is not allowed to move from cell X to cell Y at time t (because a collision with a higher-priority agent would result), delete the directed edge from state (X,t) to state (Y,t+1)
- Search the resulting state space for a time-minimal path from state (start cell, 0) to any state (goal cell, t) for all times t

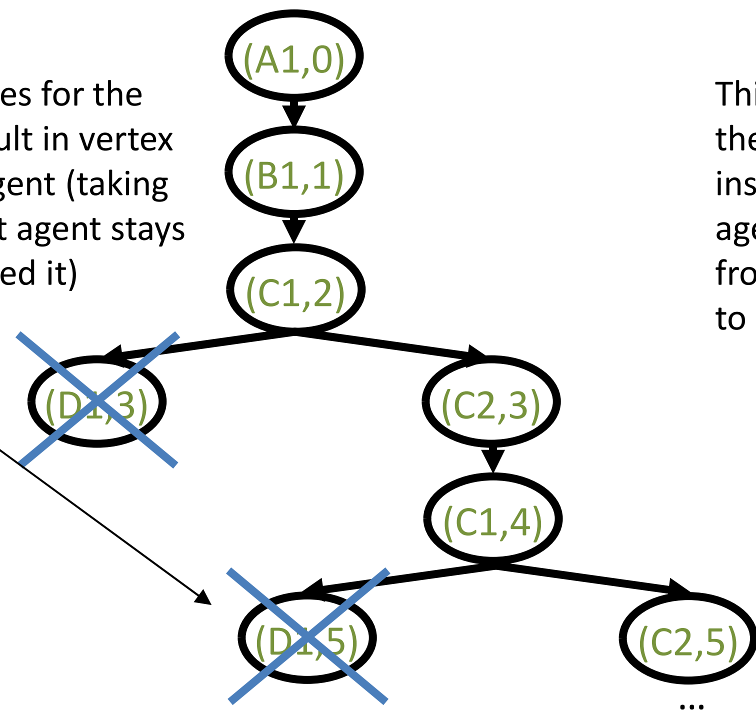
Priority-Based Search



- You could implement space (= cell)-time A* with a reservation table (specific for a particular agent) but you might not want to build it explicitly since it is often large. Rather, you never want to generate the states or edges that you would have deleted in the reservation table in the A* search tree

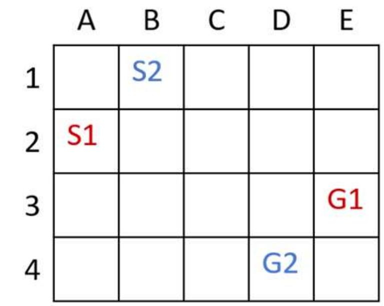
Do not generate these states for the green agent since they result in vertex collisions with the violet agent (taking into account that the violet agent stays in cell D3 once it has reached it)

The violet agent has priority 1



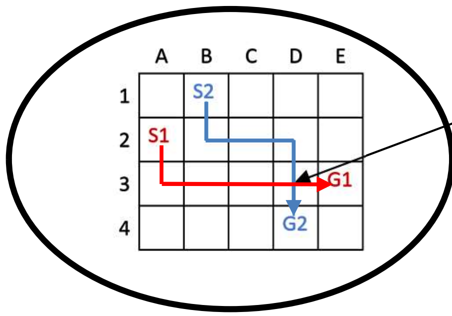
Think about how to detect that there is no path for the green agent instead of believing that the green agent can now repeatedly move from cell C1 via cell C2 back to cell C1 to eventually get to its goal cell E1

Conflict-Based Search



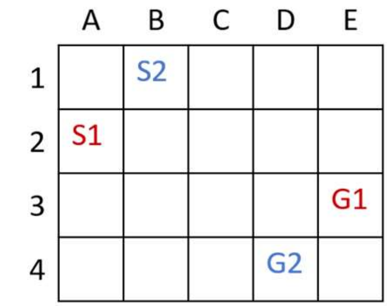
- Conflict-based search [Sharon, Stern, Felner and Sturtevant]: Optimal (or bounded-suboptimal) MAPF solver that plans for each agent independently, if possible

Find time-minimal paths for all agents independently



Conflict (here: vertex collision)

Conflict-Based Search



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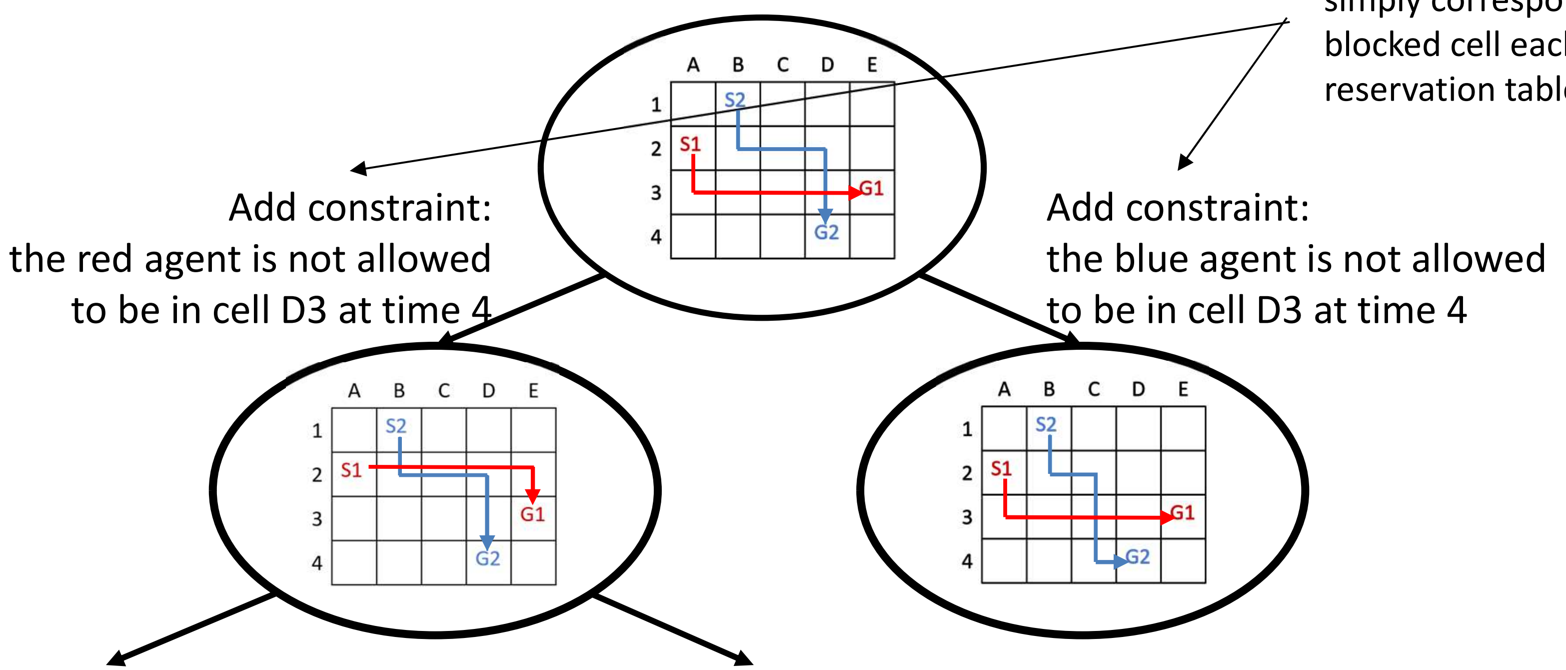
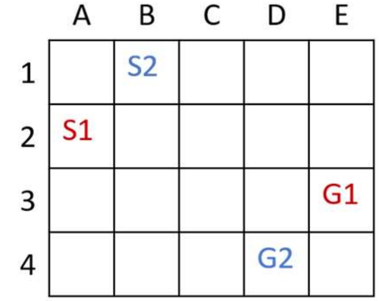
Such vertex constraints simply correspond to one blocked cell each in the reservation table



Conflict-Based Search



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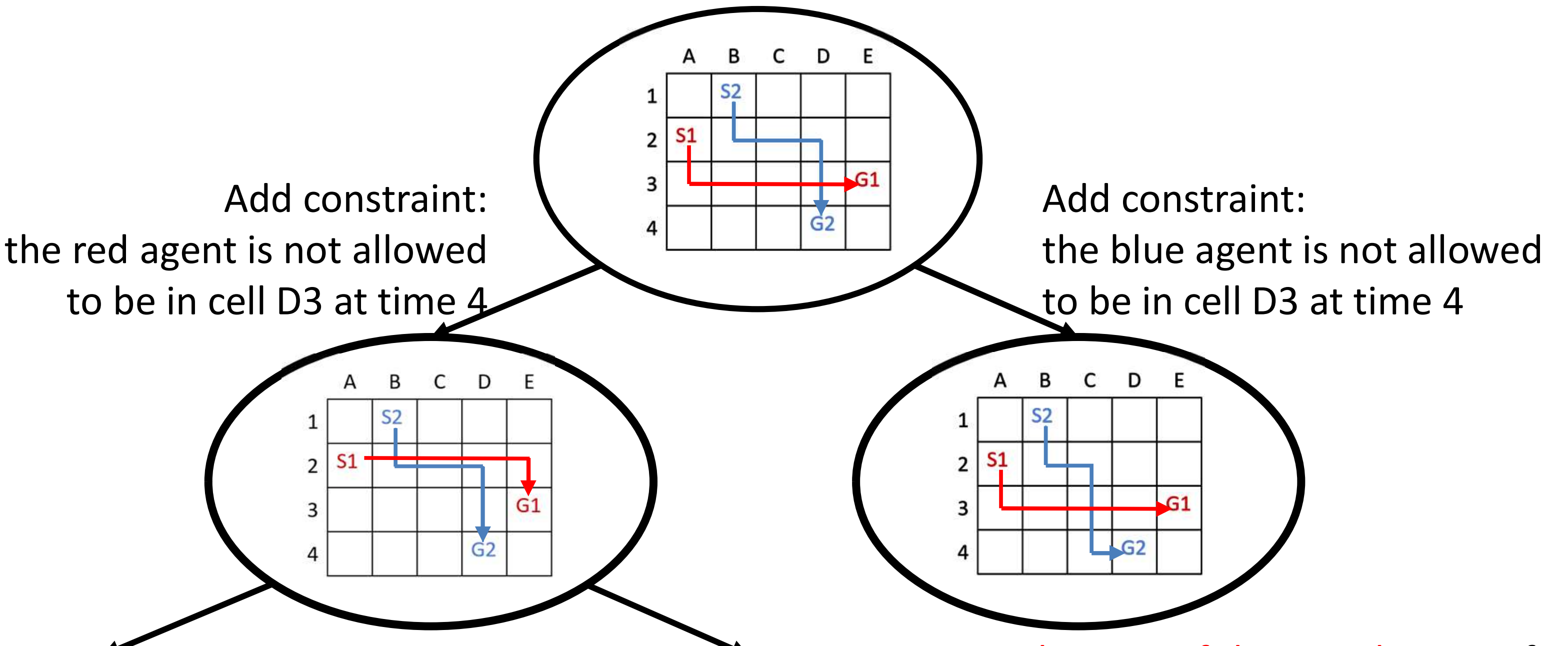
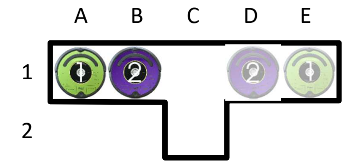


To minimize the sum of the travel times of all agents perform a best-first search on this tree with

- g = cost = sum of travel times of all agents (here: 10)
- h = 0

Conflict-Based Search



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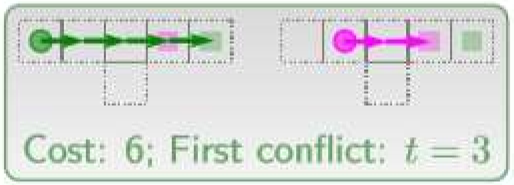
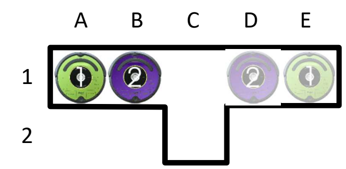


A1, B1, C1, D1, E1 B1, C1, D1

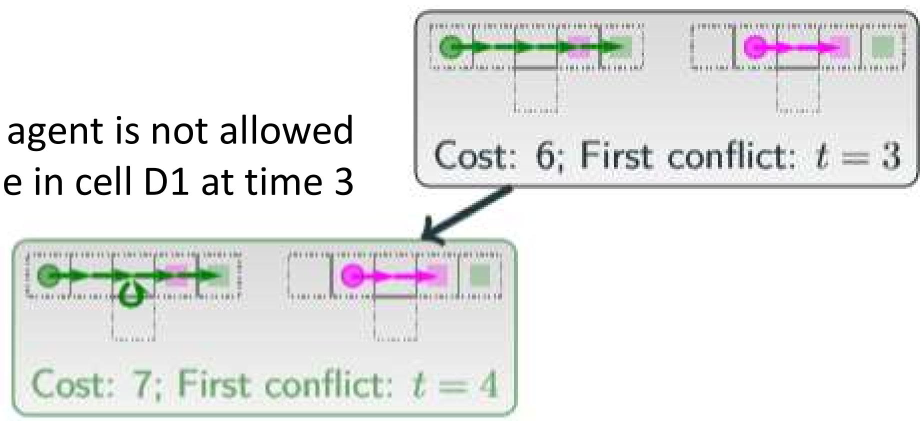
- Find time-minimal paths for both agents independently, which results in a vertex collision in cell D1 at time 3; clearly, the green agent cannot be in cell D1 at time 3 or the violet agent cannot be in cell D1 at time 3

Conflict-Based Search



- Conflict-based search [Sharon, Stern, Felner and Sturtevant]: Optimal (or bounded-suboptimal) MAPF solver that plans for each agent independently, if possible

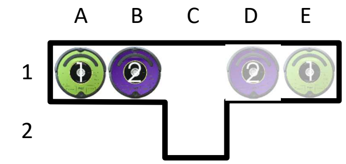
the green agent is not allowed
to be in cell D1 at time 3



A1, B1, C1, C1 (= wait), D1, E1   B1, C1, D1

- Work on the leaf node with the smallest cost; impose the vertex constraint: the green agent is not allowed to be in cell D1 at time 3; create a new child node, and replan the path of the green agent, which results in a vertex collision in cell D1 at time 4

Conflict-Based Search



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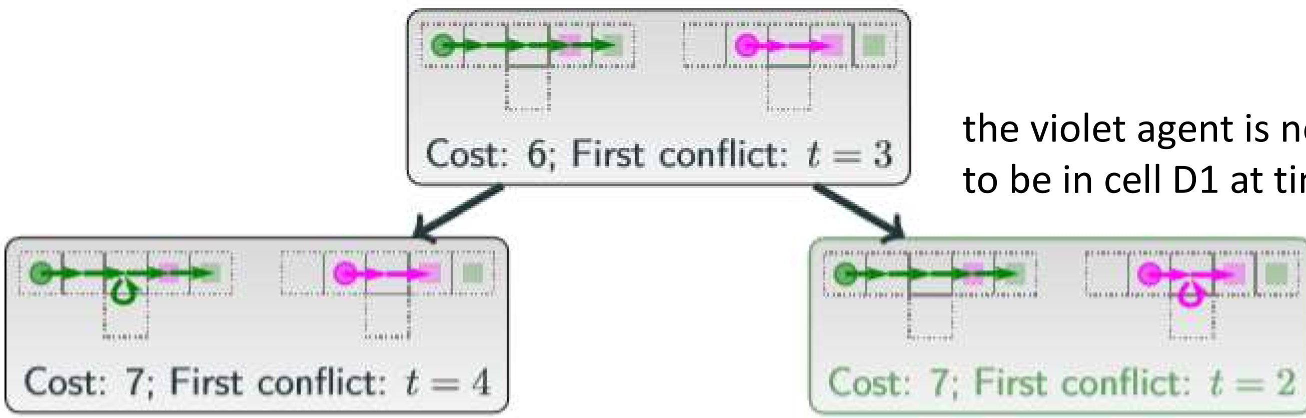
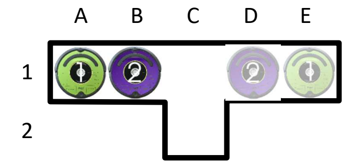


the violet agent is not allowed to be in cell D1 at time 3

A1, B1, C1, D1, E1   B1, C1, C1 (= wait), D1

- Impose also the vertex constraint: the violet agent is not allowed to be in cell D1 at time 3, create a new child node, and replan the path of the violet agent, which results in a vertex collision in cell C1 at time 2

Conflict-Based Search



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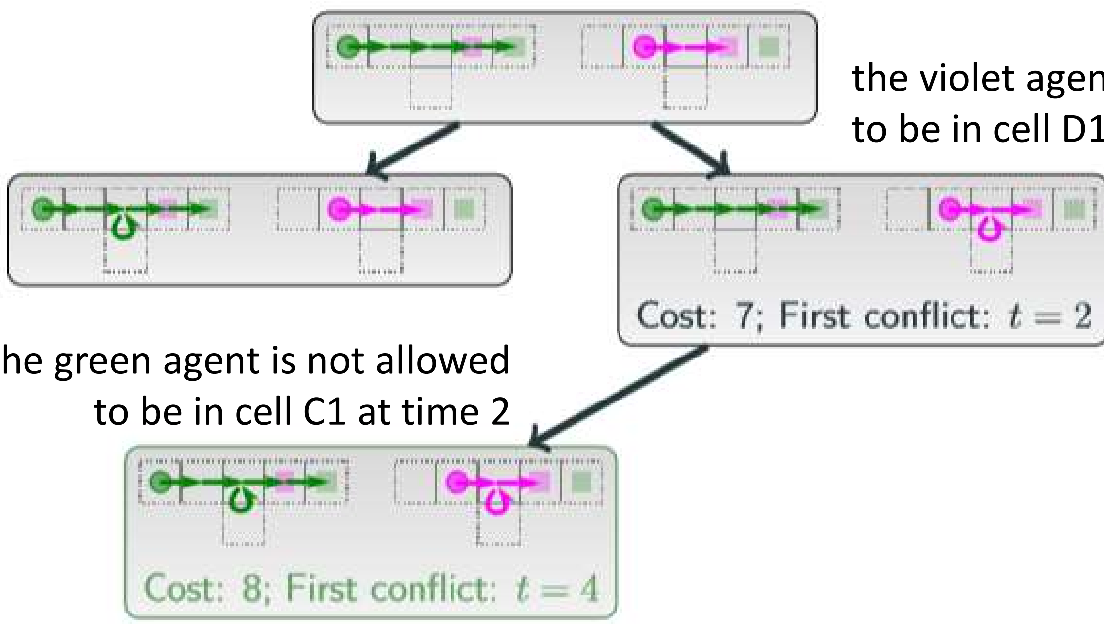


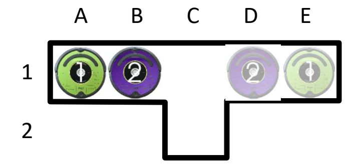
the violet agent is not allowed to be in cell D1 at time 3

the green agent is not allowed to be in cell C1 at time 2

A1, B1, C1, C1 (= wait), D1, E1   B1, C1, C1 (= wait), D1

- Work on the leaf node with the smallest cost; impose the vertex constraint: the green agent is not allowed to be in cell C1 at time 2 (in addition to the previous vertex constraint), create a new child new, and replan the path of the green agent, which results in a vertex collision in cell D1 at time 4

Conflict-Based Search



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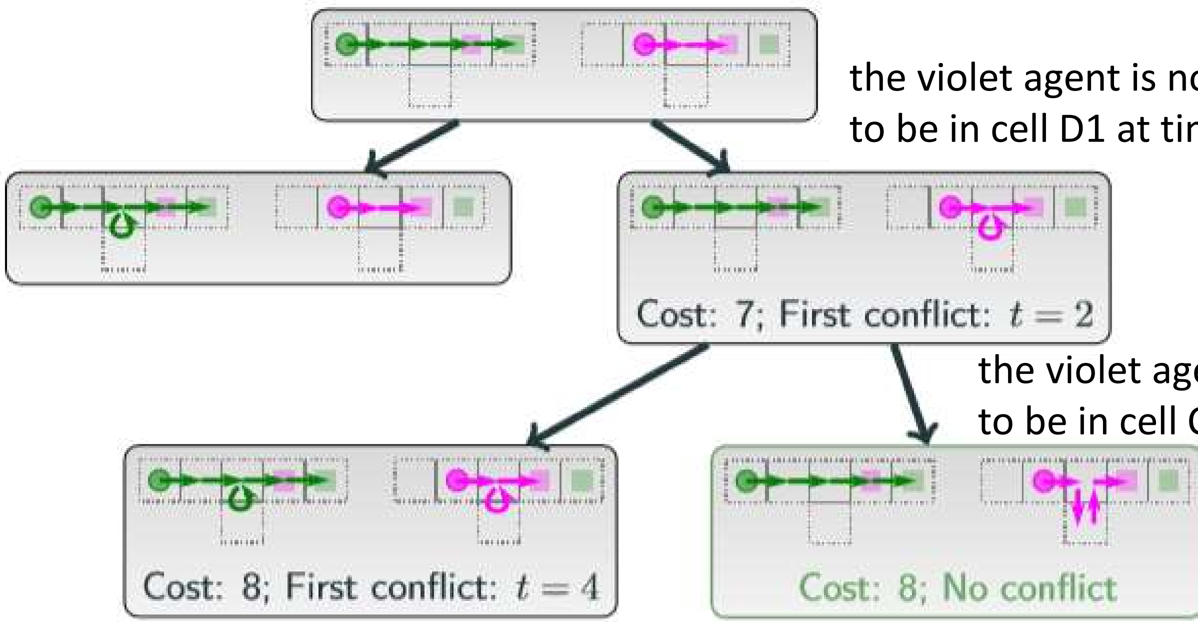


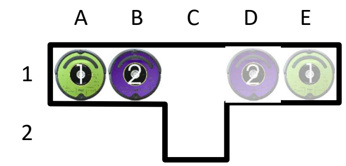
the violet agent is not allowed to be in cell D1 at time 3

the violet agent is not allowed to be in cell C1 at time 2

A1, B1, C1, D1, E1 B1, C1, C2, C1, D1

- Impose also the vertex constraint: the violet agent is not allowed to be in cell C1 at time 2 (in additional to the previous vertex constraint), work on the child node with the smallest cost, and replan the path of the violet agent, which results in no vertex or edge collisions

Conflict-Based Search



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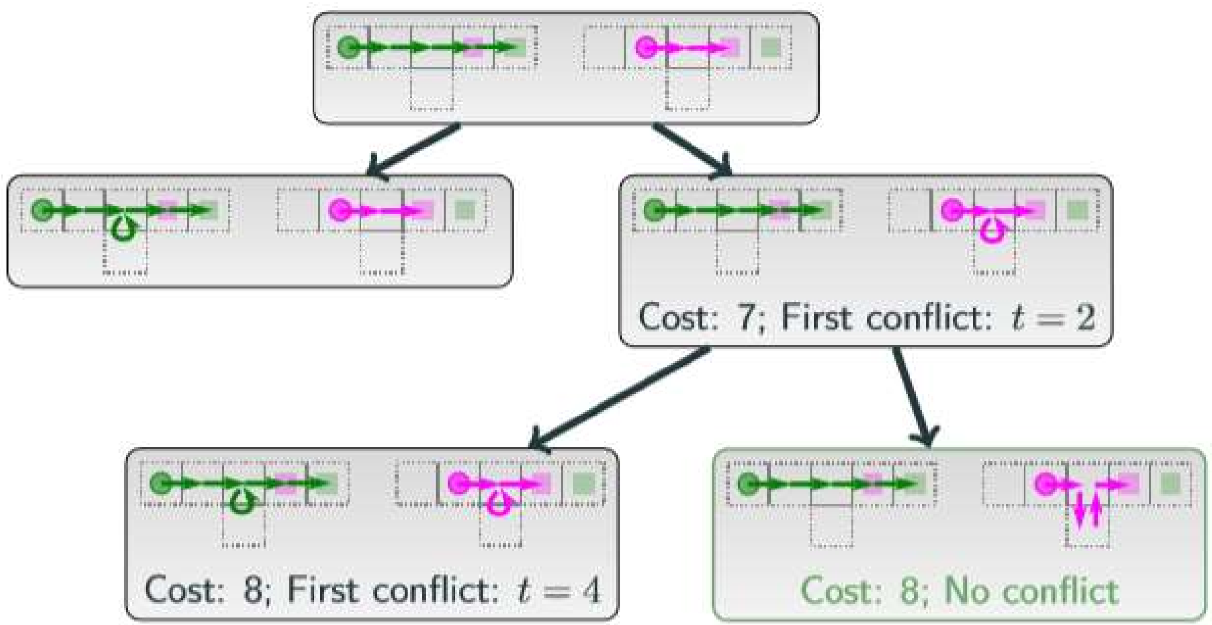
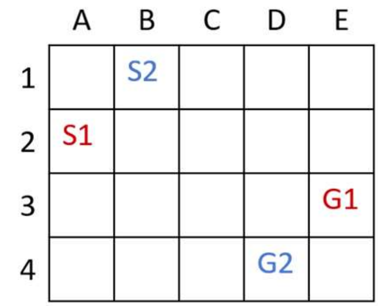


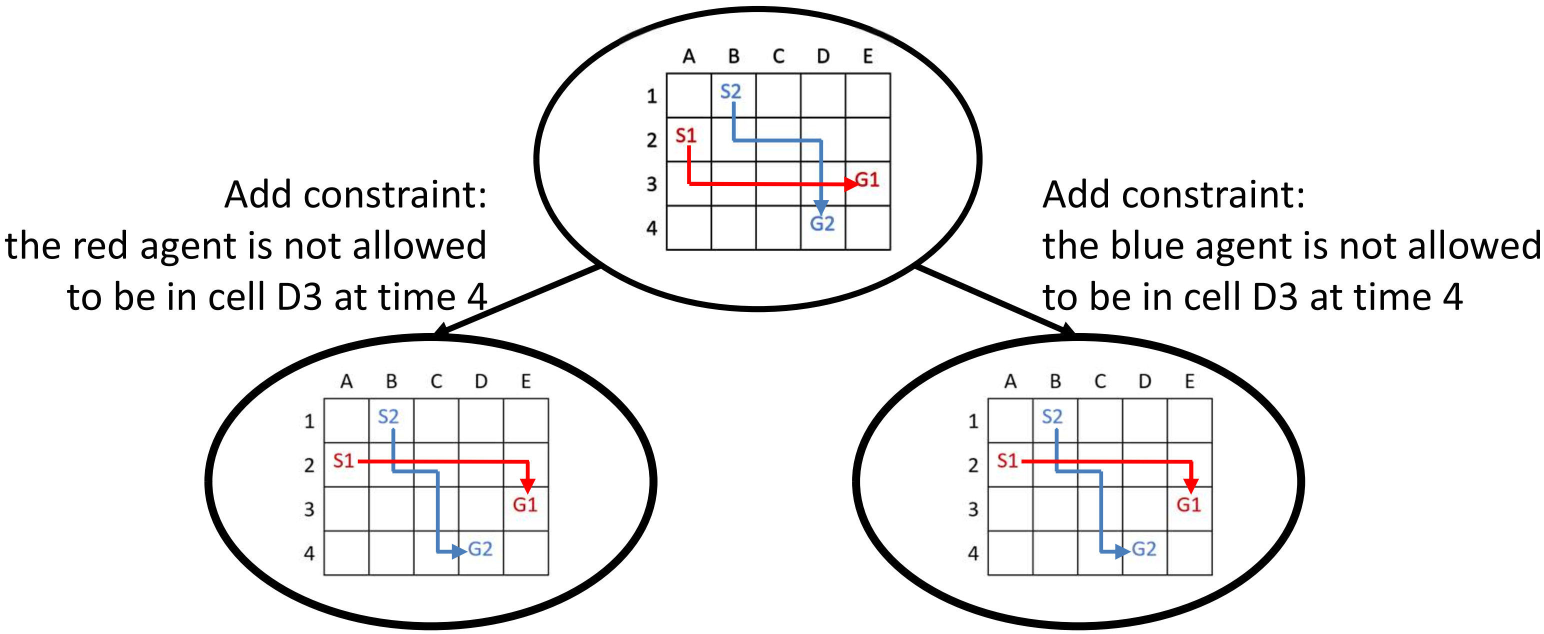
A1, B1, C1, D1, E1 B1, C1, C2, C1, D1

- Work on the leaf node with the smallest cost and terminate since this node has no vertex or edge collisions

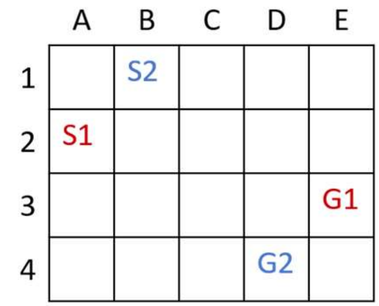
Conflict-Based Search with Disjoint Splitting



- Conflict-based search (without disjoint splitting) [Sharon, Stern, Felner and Sturtevant]: Optimal (or bounded-suboptimal) MAPF solver that plans for each agent independently, if possible

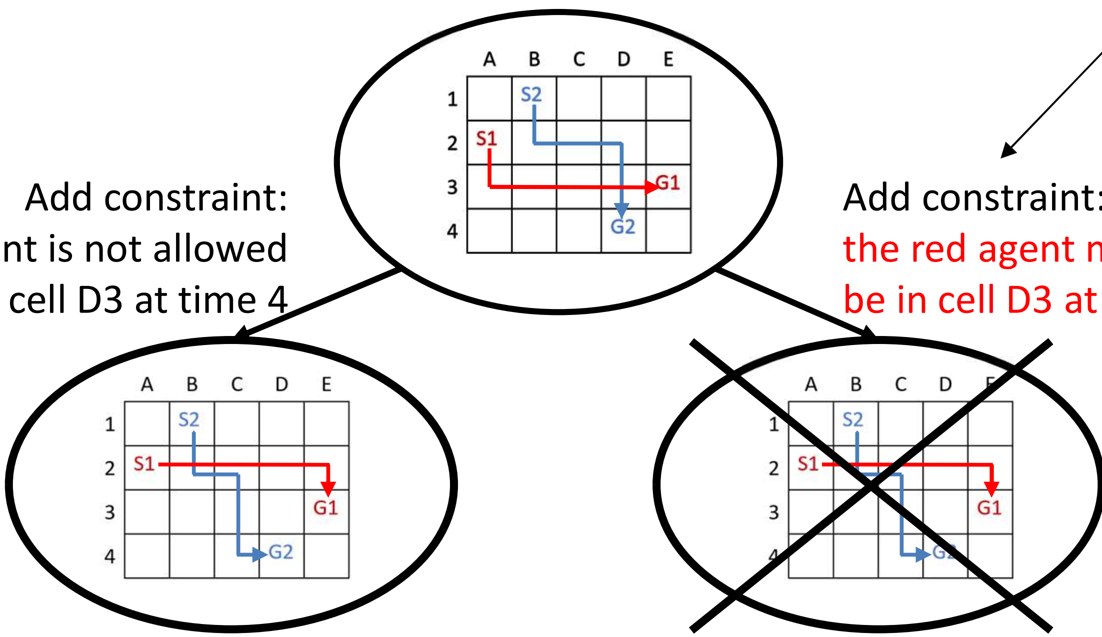


Conflict-Based Search with Disjoint Splitting



- Conflict-based search with disjoint splitting [Li, Harabor, Stuckey, Felner, Ma and Koenig]: Optimal (or bounded-suboptimal) MAPF solver that plans for each agent independently, if possible

This implies that all other agents are not allowed to be in cell D3 at time 4 – so the constraint is stricter than before!

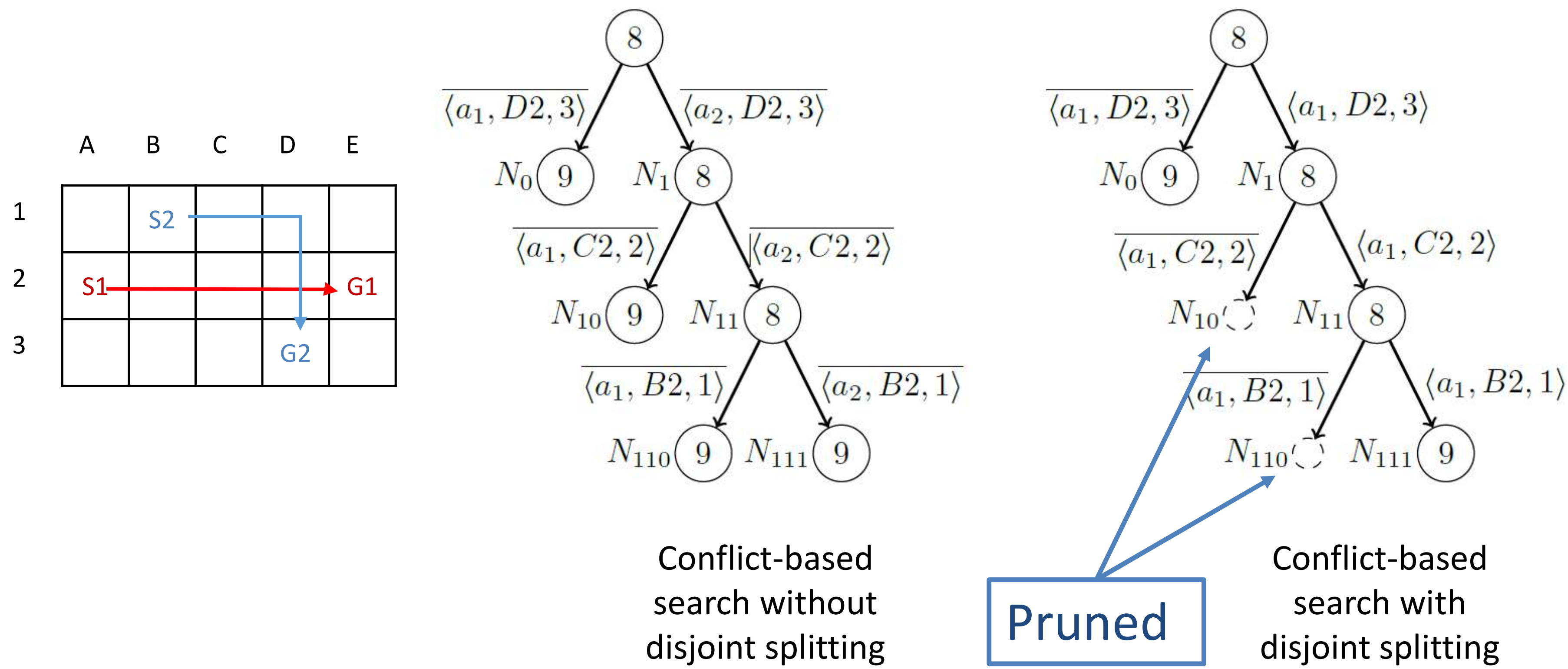


Add constraint: the red agent is not allowed to be in cell D3 at time 4

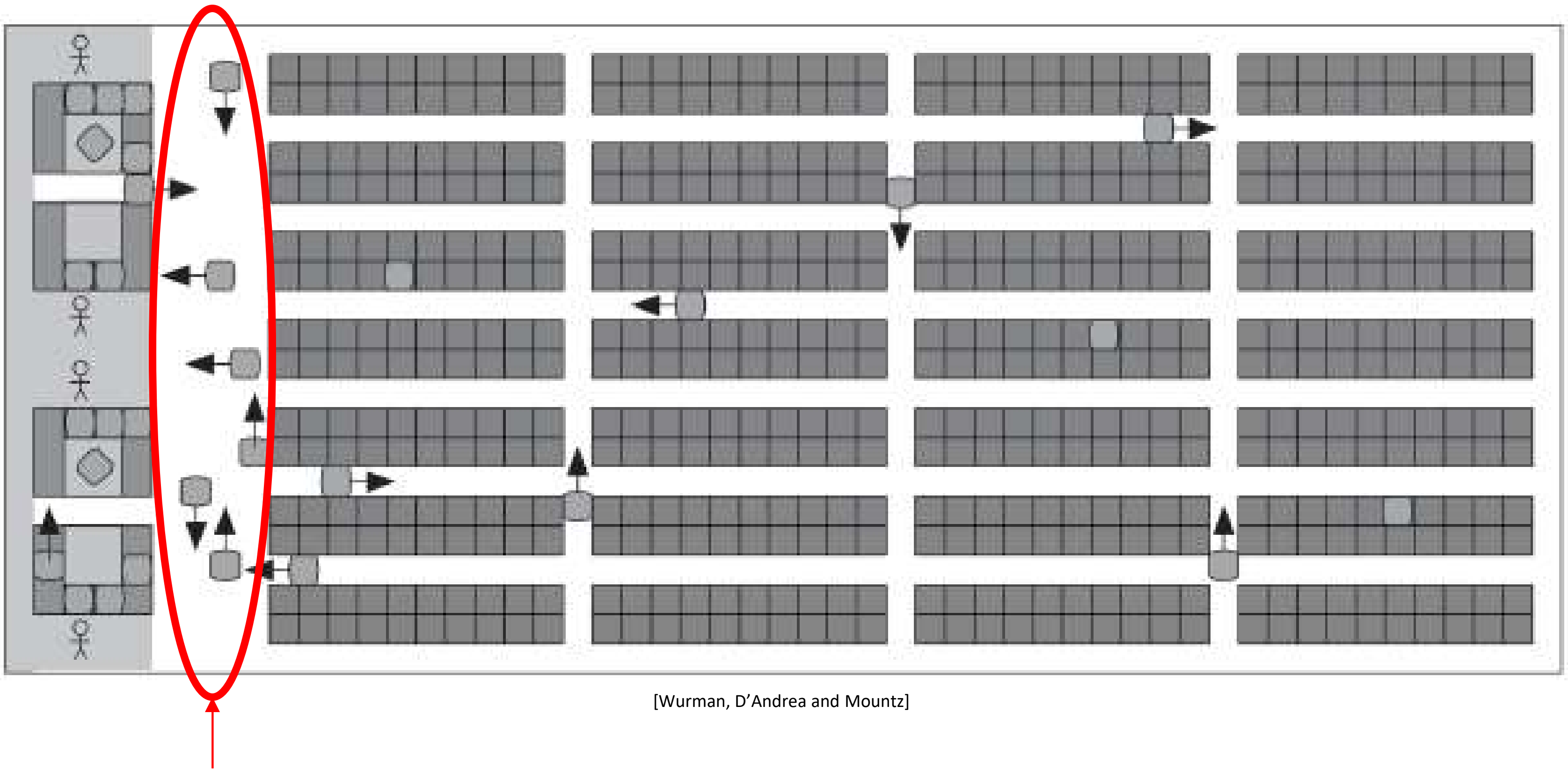
Add constraint: the red agent must be in cell D3 at time 4

Conflict-Based Search with Disjoint Splitting

- Conflict-based search with disjoint splitting: Optimal (or bounded-suboptimal) MAPF solver that plans for each agent independently, if possible



Execution of MAPF Plans



[Wurman, D'Andrea and Mountz]

Use the MAPF methods here (in a small area of high congestion but with few agents) rather than over the whole fulfillment center

Execution of MAPF Plans

- Want to learn more about multi-agent path finding?
- Visit: http://mapf.info/