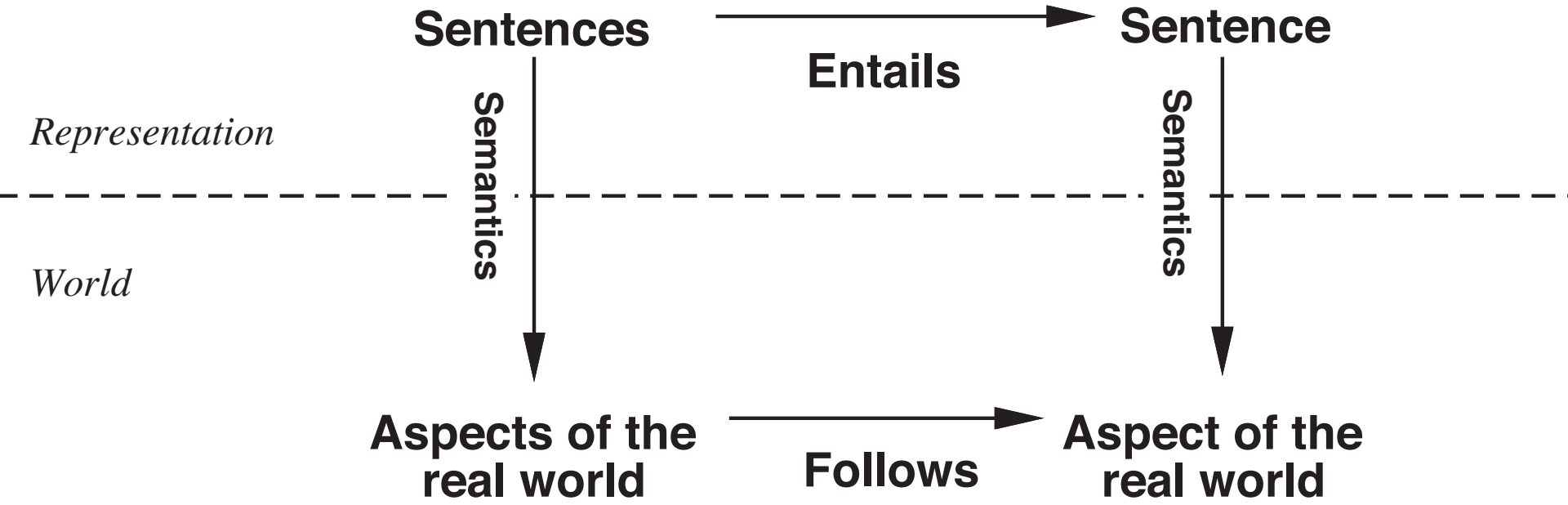
Rappresentazione e elaborazione della conoscenza: dalla sintassi (frasi della KB) alla semantica (mondo reale)



Entails: implica
Follows: conseguenza (logica)

Procedura di Inferenza/Deduzione

- "*Metodo automatico per derivare nuove frasi (formule) a partire da quelle già presenti nella base di conoscenza (KB*)"

- **Corretta** ("sound") se per ogni formula ϕ derivata da KB usando la procedura di inferenza si ha che ϕ e' una *conseguenza logica* di KB. **Completa** se vale anche il viceversa.

- **Dimostrazione** di ϕ: insieme di operazioni eseguite dalla procedura di inferenza (corretta) per derivare ϕ

- **Teoria della dimostrazione** ("proof theory"): è relativa ad un longuaggio di KR. Specifica un insieme di passi di ragionamento/inferenza che sono corretti

Alcuni Linguaggi di KR (Knowledge Representation)

- Un linguaggio di KR deve essere **conciso**, **espressivo** (il più possibile), **non ambiguo** e indipendente dal contesto, **efficace** (esiste una procedura di inferenza corretta ed implementabile)

- **Linguaggio naturale**: espressivo, ma ambiguo, non conciso

- **Linguaggio di programmazione**: preciso, strutturato, ma poco espressivo

- **Logica**: precisa, concisa, expressiva

Logica e KR

- Varie logiche, differenti assunzioni ontologiche e epistemologiche (esempi)
- Una logica è descritta da sintassi e semantica
- Vedremo Logica Proposizionale e Logica del I Ordine (Calcolo dei Predicati) in dettaglio

Assunzioni Ontologiche/Epistemiologiche nei Linguaggi di KR

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief $\in [0, 1]$
Fuzzy logic	facts with degree of truth $\in [0, 1]$	known interval value

Figure 8.1 Formal languages and their ontological and epistemological commitments.

Sintassi logica proposizionale

$$
\begin{aligned}
\mathit{Sentence} &\rightarrow \mathit{AtomicSentence} \mid \mathit{ComplexSentence} \\
\mathit{AtomicSentence} &\rightarrow \mathit{True} \mid \mathit{False} \mid P \mid Q \mid R \mid \ldots \\
\mathit{ComplexSentence} &\rightarrow (\ \mathit{Sentence}\ ) \mid [\ \mathit{Sentence}\ ] \\
&\mid \neg\ \mathit{Sentence} \\
&\mid \mathit{Sentence} \wedge \mathit{Sentence} \\
&\mid \mathit{Sentence} \vee \mathit{Sentence} \\
&\mid \mathit{Sentence} \Rightarrow \mathit{Sentence} \\
&\mid \mathit{Sentence} \Leftrightarrow \mathit{Sentence} \\
\text{OPERATOR PRECEDENCE} &: \neg, \wedge, \vee, \Rightarrow, \Leftrightarrow
\end{aligned}
$$

Figure 7.7 A BNF (Backus–Naur Form) grammar of sentences in propositional logic, along with operator precedences, from highest to lowest.

Semantica operatori logici

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

Figure 7.8 Truth tables for the five logical connectives. To use the table to compute, for example, the value of $P \vee Q$ when P is true and Q is false, first look on the left for the row where P is *true* and Q is *false* (the third row). Then look in that row under the $P \vee Q$ column to see the result: *true*.

Semantica della Logica

- ciò che la formula asserisce sul mondo = significato della frase
- Dipende da una particolare **interpretazione**: Una formula è **vera rispetto ad una interpretazione** se ciò che rappresenta è vero nel mondo.

  Es: "*il papa è in Olanda*" ... → "*il microfilm è nel lampadario*"
- **Principio di composizionalità**: il significato di una formula è una funzione del significato delle sue parti (si per la logica!)
- **Formula valida (tautologia):** vera per qualsiasi interpretazione ("*piove* ***o*** *non piove*". Altri esempi meno ovvi dopo
- **Formula soddisfacibile**: esiste una interpretazione per cui la formula è vera ("*piove* ***e*** *non piove*"), **insoddifacibile** altrimenti ("*piove o fa-caldo*")
- Ogni formula tautologica è soddisfacibile (no vicerersa)

Modelli (logica proposizionale)

- Un **Modello** per una F.B.F ϕ è una intepretazione (associazione) dei simboli proposizionali di ϕ in valori di verità (V,F) per i quali ϕ è vero.
- Modello per ϕ: "mondo" in cui ϕ è vero rispetto ad una certa interpretazione
- Esempio, modelli di "P $\Rightarrow$ not Q": [F,F], [F,V], [V,F]
- Servono per definire il concetto (**semantico**) di **conseguenza logica**: Se l'insieme di tutti i modelli di una KB di F.B.F. sono anche modelli di ϕ, allora ϕ è una conseguenza logica di KB

KB = {"piove", "piove $\Rightarrow$ bagnato"} , ϕ = "bagnato"

Validity and Unsatisfiability

A sentence is **valid** if it is true in **all** interpretatioms,

e.g., *True*, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the **Deduction Theorem**:

$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is **unsatisfiable** if it is true in **no** interpretation

e.g., $A \wedge \neg A$

Logical implication (entailment) is connected to unsatisfiability via the following: $\boldsymbol{KB \models \alpha}$ $((KB \Rightarrow \alpha)$ is valid)

if and only if $(KB \wedge \neg \alpha)$ is unsatisfiable

Dimostrazioni e Inferenza (ragionamento deduttivo)

- **Dimostrazione (derivazione) logica** di una formula **f**: sequenza di applicazioni di ***regole di inferenza*** a partire dalle F.B.F. nella KB fino a generare il **teorema f**

- **Procedura di inferenza**: procedura per costruire dimostrazioni logiche attraverso l'individuazione di appropriate *sequenze di applicazioni di regole di inferenza*

- **Regole di inferenza**: *Modus ponens*, *and-elimination*, *and-introduction*, *or-introduction*, *double-negation*, *unit resolution*, *resolution*

- Richiedono formule in *formato normalizzato*

- Esempi di dimostrazione

Metodi semantici (model checking)

- Non usano procedure di inferenza, ma modelli (dimostriamo la conseguenza logica).
- Metodo basato su **tabelle di verità** (esempi)

KB = {A, not B, (A or B $\Rightarrow$ C)} $\boldsymbol{KB \models C\ ?}$

KB = {A, not B, (A or B $\Rightarrow$ C)} $\boldsymbol{KB \models (B \Rightarrow C)\ ?}$

- Complessità temporale e spaziale *esponenziale*.....

Altro Metodo Semantico

Φ *è una conseguenza logica di una KB consistente (soddisfacibile) se e solo se KB' = KB + NOT Φ è insoddifacibile (non ha modelli).*

- Formulo il problema come **CSP Booleano** (**SAT**).
  Se trovo un modello per KB', allora è soddisfacibile e Φ non è una conseguenza logica di KB (altrimenti lo è)

- Complessità esponenziale nel caso generale, polinomiale per 2SAT e per *formule di Horn*

Resolution

Conjunctive Normal Form (CNF)

conjunction of disjunctions of literals (= conjunction of clauses)

E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

- Resolution inference rule (for CNF):
-

$$\frac{\ell_i \vee \ldots \vee \ell_k, \qquad m_1 \vee \ldots \vee m_n}{\ell_i \vee \ldots \vee \ell_{i-1} \vee \ell_{i+1} \vee \ldots \vee \ell_k \vee m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n}$$

where ℓ_i and m_j are *complementary* literals.

Resolution is sound and complete for propositional logic

Resolution

Implicative version of the rule

$$\frac{\neg(l_i \vee \ldots \vee l_{i-1} \vee l_{i+1} \vee \ldots \vee l_k) \Rightarrow l_i \qquad \neg m_j \Rightarrow (m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n)}{\neg(l_i \vee \ldots \vee l_{i-1} \vee l_{i+1} \vee \ldots \vee l_k) \Rightarrow (m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n)}$$

Conversion to CNF

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.
   $(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$

4. Apply distributivity law ($\wedge$ over $\vee$) and flatten:

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

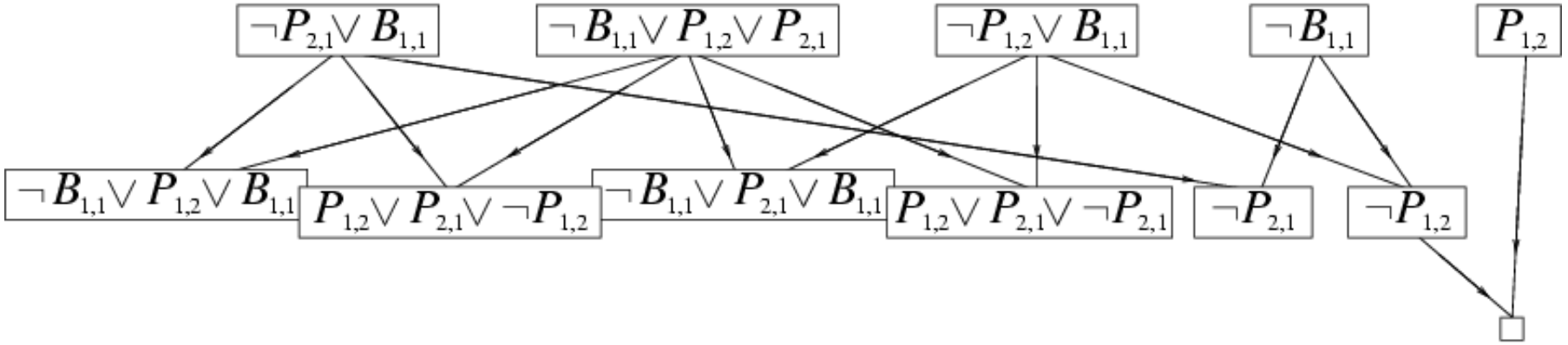
Some Logical Equivalences

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) && \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) && \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) && \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) && \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha && \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) && \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) && \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) && \text{De Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) && \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) && \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) && \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Figure 7.11 Standard logical equivalences. The symbols α, β, and γ stand for arbitrary sentences of propositional logic.

Resolution example

- $KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$
- $\alpha = \neg P_{1,2}$
-



Completezza della risoluzione

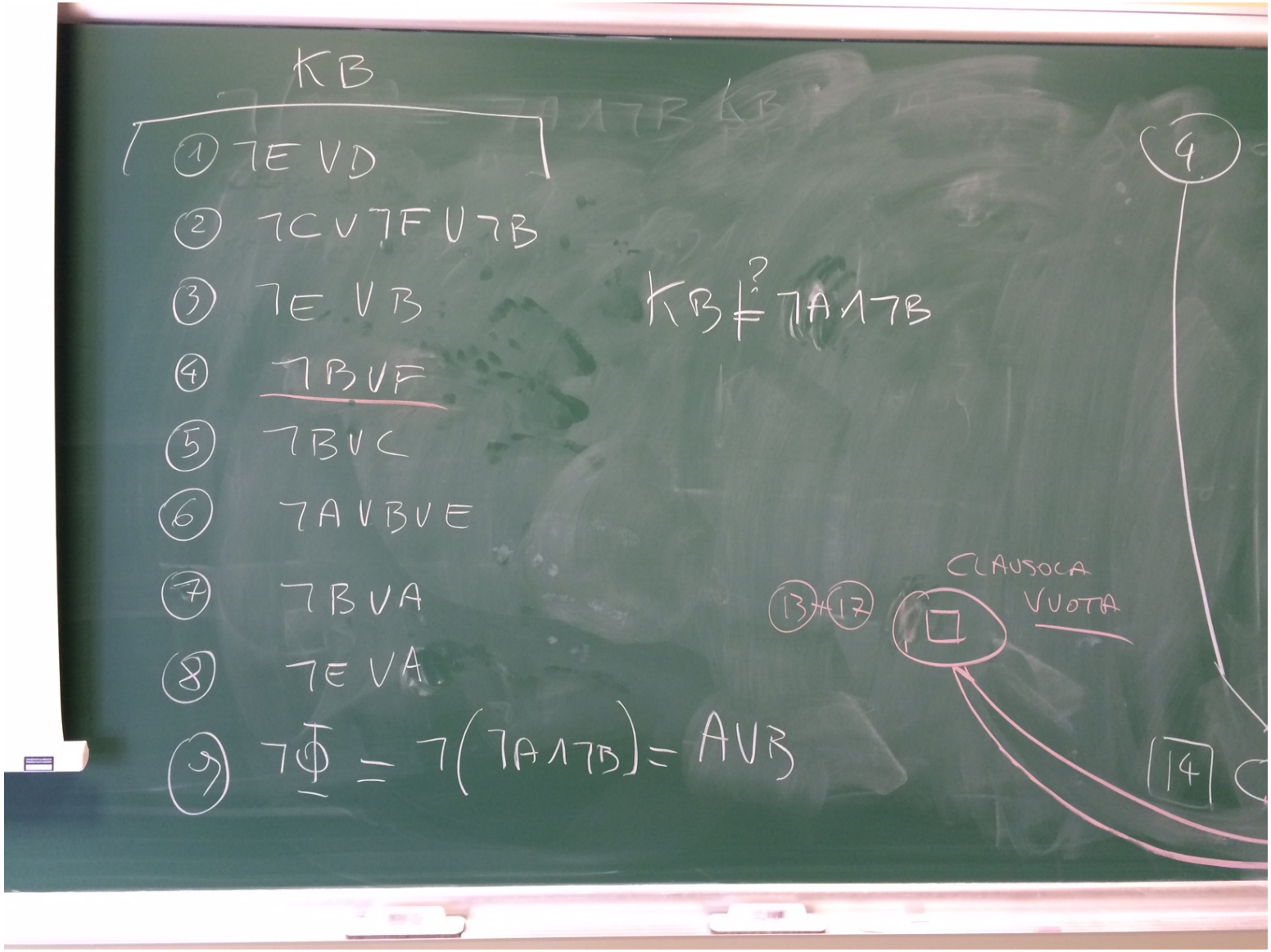
- *Una procedura di inferenza che applica la regola di risoluzione con una **opportuna strategia** è **completa** per dimostrare, attraverso il **metodo di refutazione, qualsiasi formula** della logica proposizionale*
- **Strategia di risoluzione**: come scelgo le 2 clausole da risolvere
- La KB congiunta con la formula negata devono essere in **formato CNF** (congiunzione/insieme di clausole)
- Esempio: Se KB = {A} non posso derivare “A or B” con la risoluzione, ma posso dimostrare “A or B” attraverso il metodo di refutazione….

Resolution algorithm

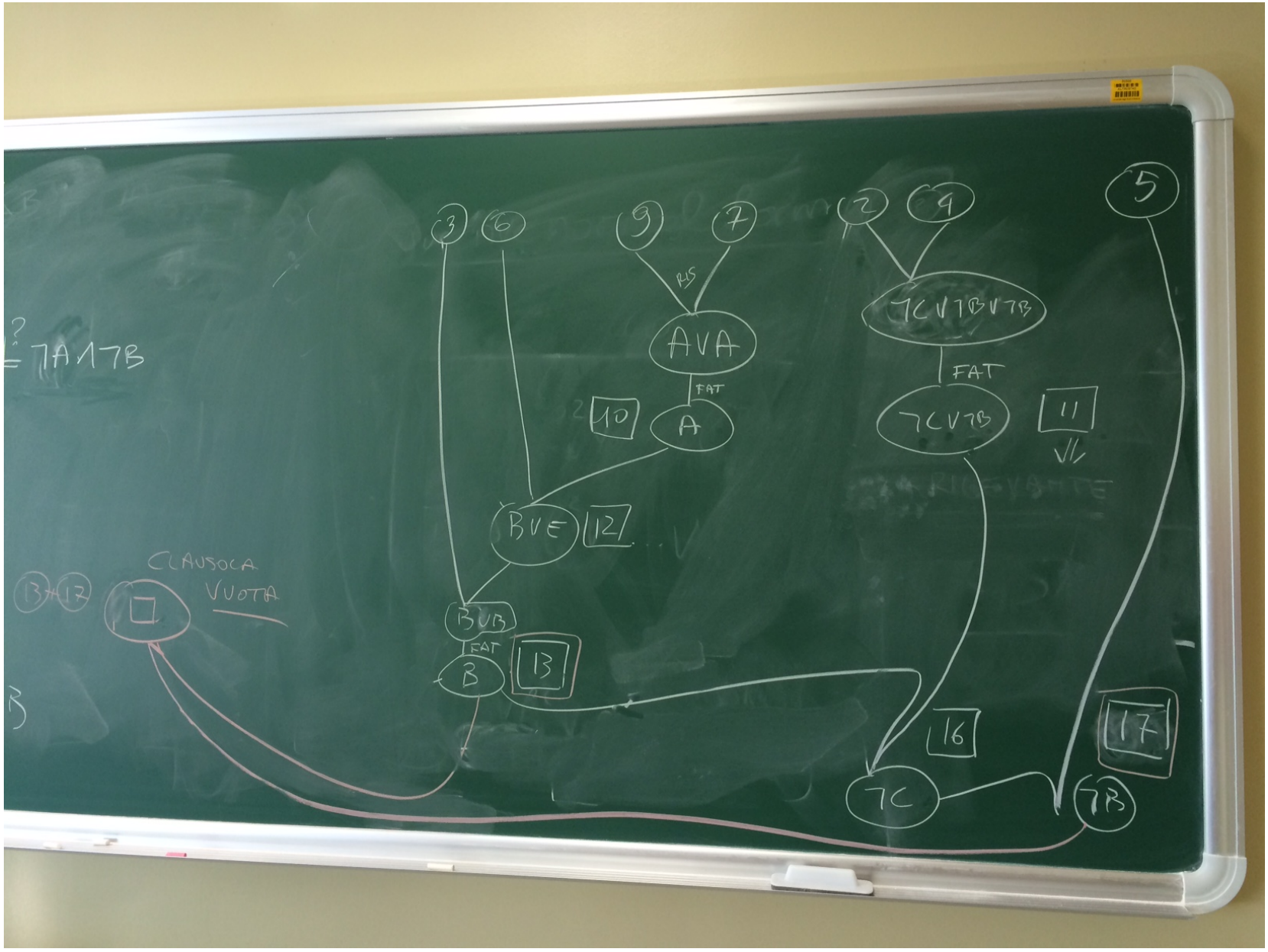
- Proof by contradiction, i.e., show $KB \wedge \neg \alpha$ unsatisfiable

```
function PL-RESOLUTION(KB, α) returns true or false
    clauses ← the set of clauses in the CNF representation of KB ∧ ¬α
    new ← { }
    loop do
        for each C_i, C_j in clauses do
            resolvents ← PL-RESOLVE(C_i, C_j)
            if resolvents contains the empty clause then return true
            new ← new ∪ resolvents
        if new ⊆ clauses then return false
        clauses ← clauses ∪ new
```

Resolution example (2)



Solution example 2 (refutation graph)



CNF, Horn, Clausole Definite

$$
\begin{aligned}
\mathit{CNFSentence} &\rightarrow \mathit{Clause}_1 \wedge \cdots \wedge \mathit{Clause}_n \\
\mathit{Clause} &\rightarrow \mathit{Literal}_1 \vee \cdots \vee \mathit{Literal}_m \\
\mathit{Literal} &\rightarrow \mathit{Symbol} \mid \neg \mathit{Symbol} \\
\mathit{Symbol} &\rightarrow P \mid Q \mid R \mid \ldots \\
\mathit{HornClauseForm} &\rightarrow \mathit{DefiniteClauseForm} \mid \mathit{GoalClauseForm} \\
\mathit{DefiniteClauseForm} &\rightarrow (\mathit{Symbol}_1 \wedge \cdots \wedge \mathit{Symbol}_l) \Rightarrow \mathit{Symbol} \\
\mathit{GoalClauseForm} &\rightarrow (\mathit{Symbol}_1 \wedge \cdots \wedge \mathit{Symbol}_l) \Rightarrow \mathit{False}
\end{aligned}
$$

Figure 7.14 A grammar for conjunctive normal form, Horn clauses, and definite clauses. A clause such as $A \wedge B \Rightarrow C$ is still a definite clause when it is written as $\neg A \vee \neg B \vee C$, but only the former is considered the canonical form for definite clauses. One more class is the k-CNF sentence, which is a CNF sentence where each clause has at most k literals.

Forward and backward chaining

- Horn Form (restricted)

  KB = conjunction of Horn clauses (DEFINITE CLAUSES)

  – Horn clause =
    - proposition symbol; or
    - (conjunction of symbols) $\Rightarrow$ symbol

  – E.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$

$\vee P_{2,1}) \wedge \neg B_{1,1}$

- Modus Ponens (for Horn Form): complete for Horn KBs

$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \ldots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

- Can be used with forward chaining or backward chaining.
- These algorithms are very natural and run in **linear** time!

Esercizi

$\neg P \vee Q$

$\neg L \vee \neg M \vee P$

$\neg B \vee \neg L \vee M$

$\neg A \vee \neg P \vee L$

$\neg A \vee \neg B \vee L$

A

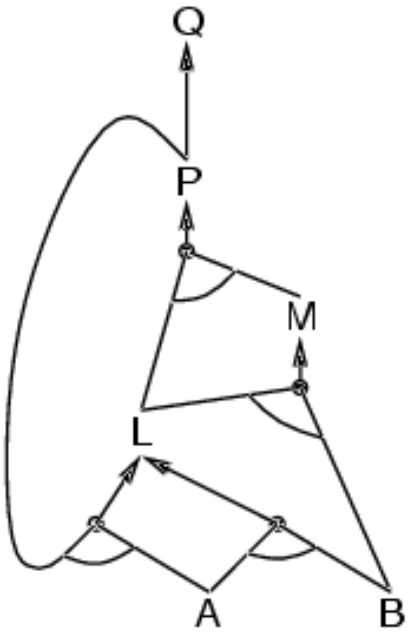
B

1. Sono clausole di Horn? Sono definite?
2. Convertite in forma implicative (come vuole modus ponens)
3. Costruire l'ipergrafo AND-OR che rappresenta KB
4. Dimostrare se KB $\models$ Q usando
   - risoluzione con strategie: **ampiezza**, clausula **unitaria** positiva, **lineare**
   - forward chaining (dopo trasformazione in clausole definite implicative)

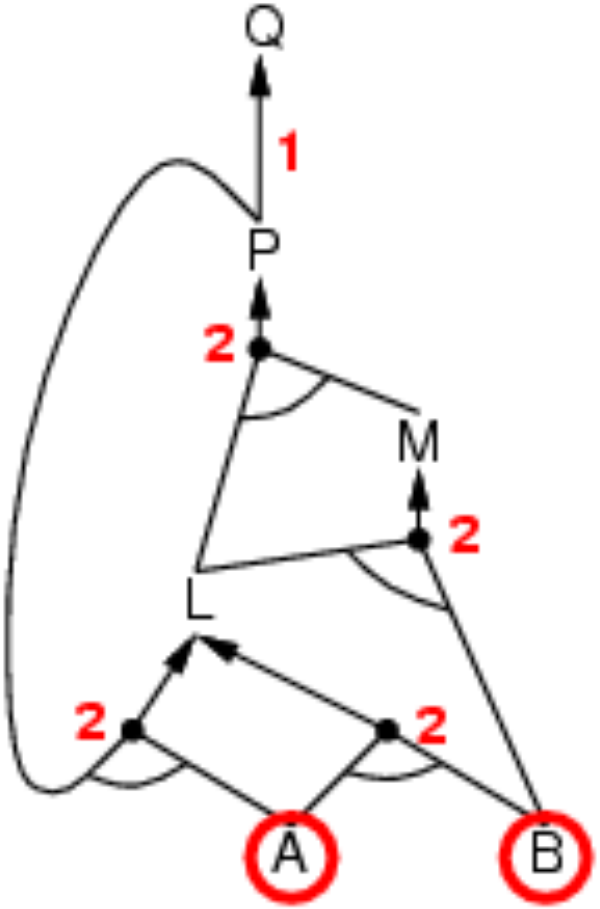
Forward chaining

- Idea: fire any rule whose premises are satisfied in the *KB*,
  - add its conclusion to the *KB*, until query is found

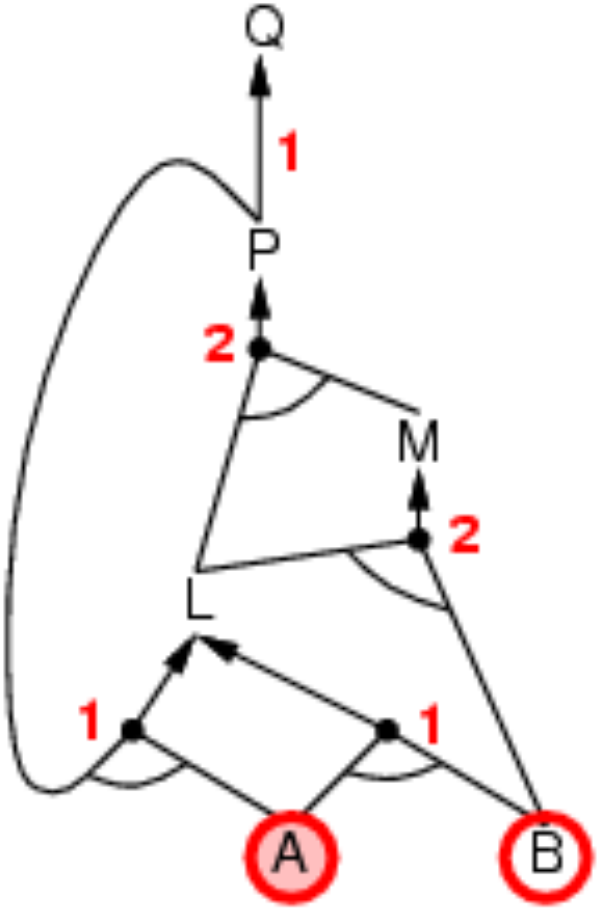
$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



Forward chaining example



Forward chaining example



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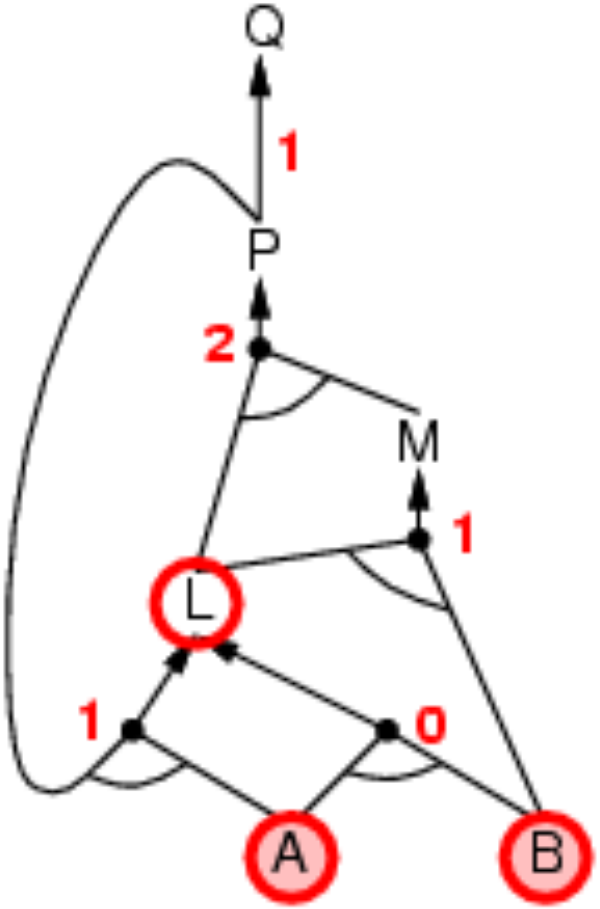

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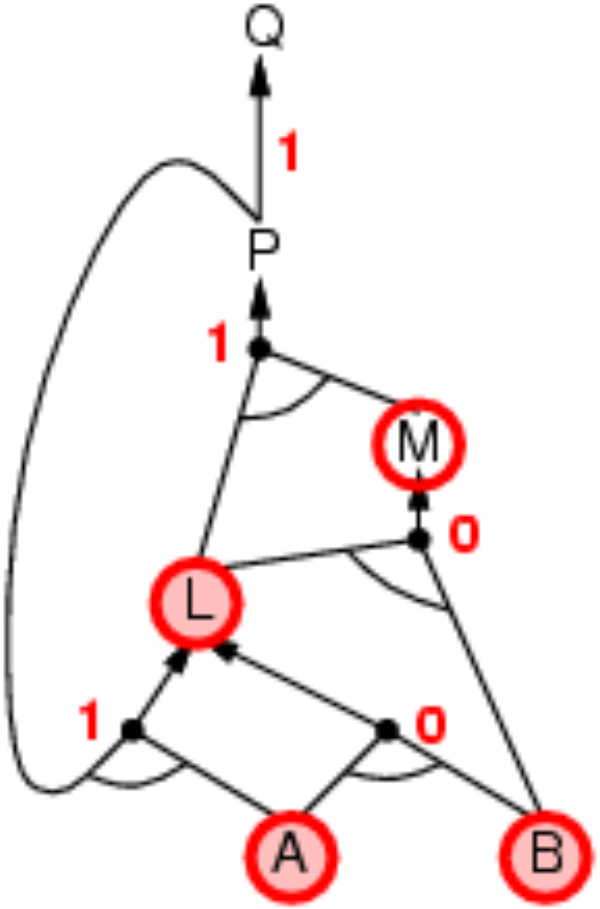
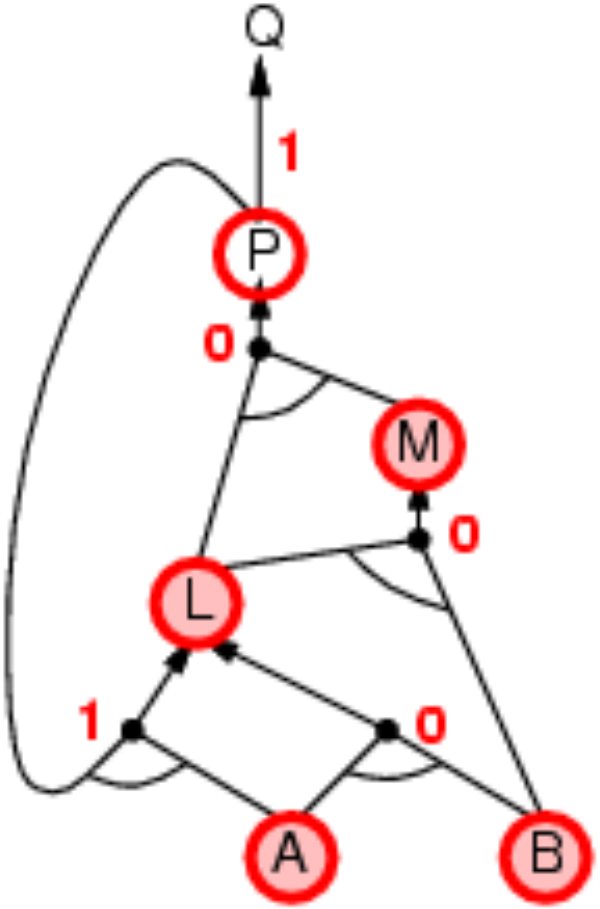


Forward chaining example



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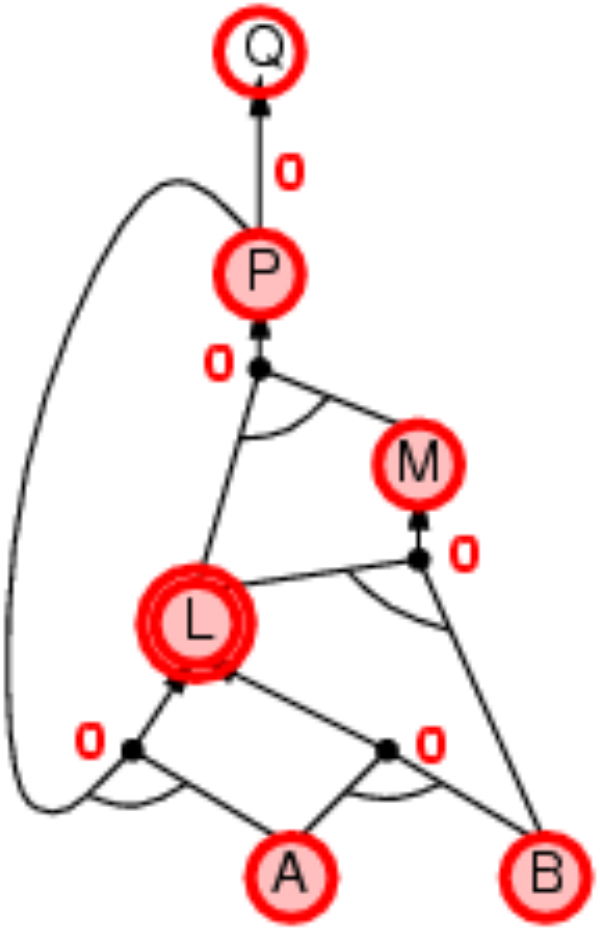

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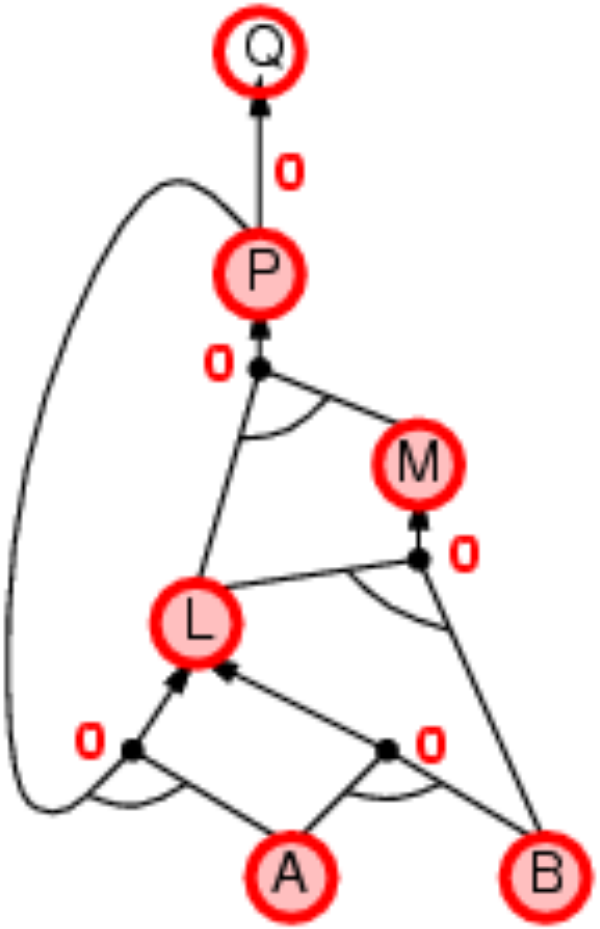
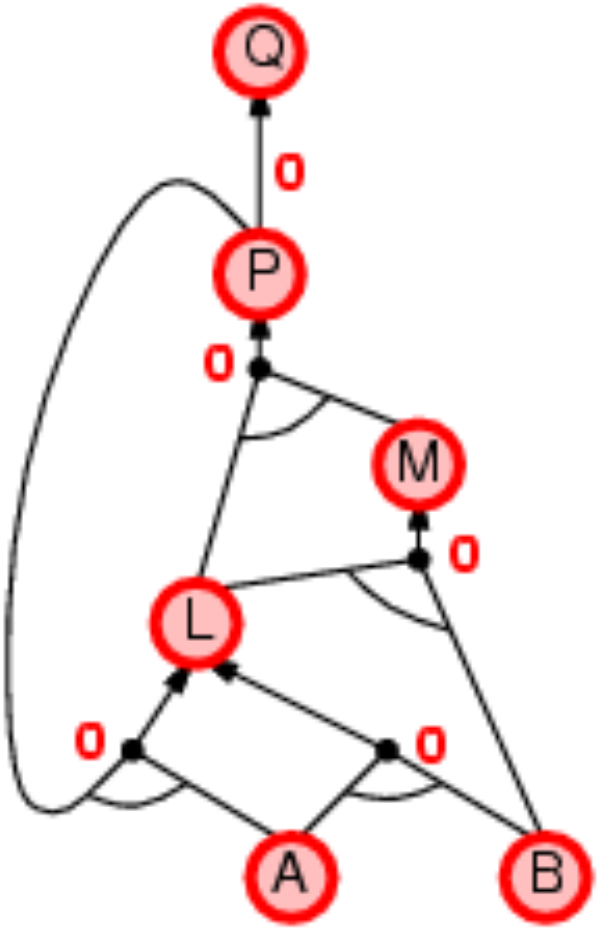


Forward chaining example



Forward chaining algorithm

```
function PL-FC-ENTAILS?(KB, q) returns true or false
  local variables: count, a table, indexed by clause, initially the number of premises
                   inferred, a table, indexed by symbol, each entry initially false
                   agenda, a list of symbols, initially the symbols known to be true

  while agenda is not empty do
      p ← POP(agenda)
      unless inferred[p] do
          inferred[p] ← true
          for each Horn clause c in whose premise p appears do
              decrement count[c]
              if count[c] = 0 then do
                  if HEAD[c] = q then return true
                  PUSH(HEAD[c], agenda)
  return false
```

- Forward chaining is sound and complete for Horn KB

Proof of completeness

FC derives every atomic sentence that is entailed by *KB*

1. FC reaches a fixed point where no new atomic sentences are derived
2. Consider the final state as a model *m*, assigning true/false to symbols
3. Every clause in the original *KB* is true in *m*
   (if $a_1 \wedge \ldots \wedge a_k \Rightarrow b$ is *false* in *m* FC has not reached fixed point)
4. Hence *m* is a model of *KB*
5. If $KB \models q$, *q* is true in every model of *KB*, including *m*

Perciò è impossibile che FC **non** derivi q; se così fosse, allora per costruzione di **m** esisterebbe un modello (m) di KB in cui $q = false$ e, per definizione di $\models$, **m** dovrebbe essere anche modello di *q*, che è impossibile

Backward Chaining (BC)

Idea: work backwards from the query *q*:

to prove *q* by BC,

check if *q* is known already, or

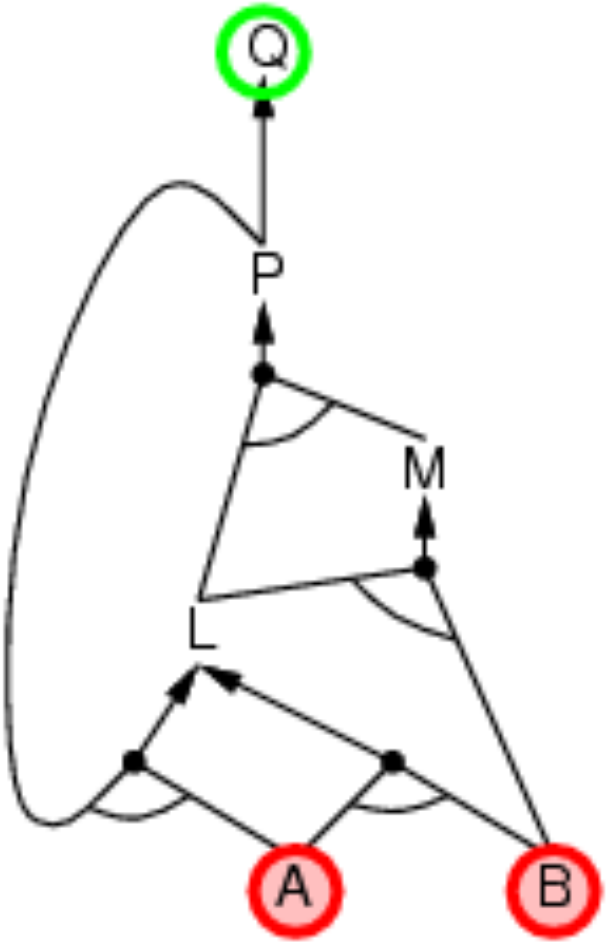
prove by BC all premises of some rule concluding *q*

Avoid loops: check if new subgoal is already on the goal stack

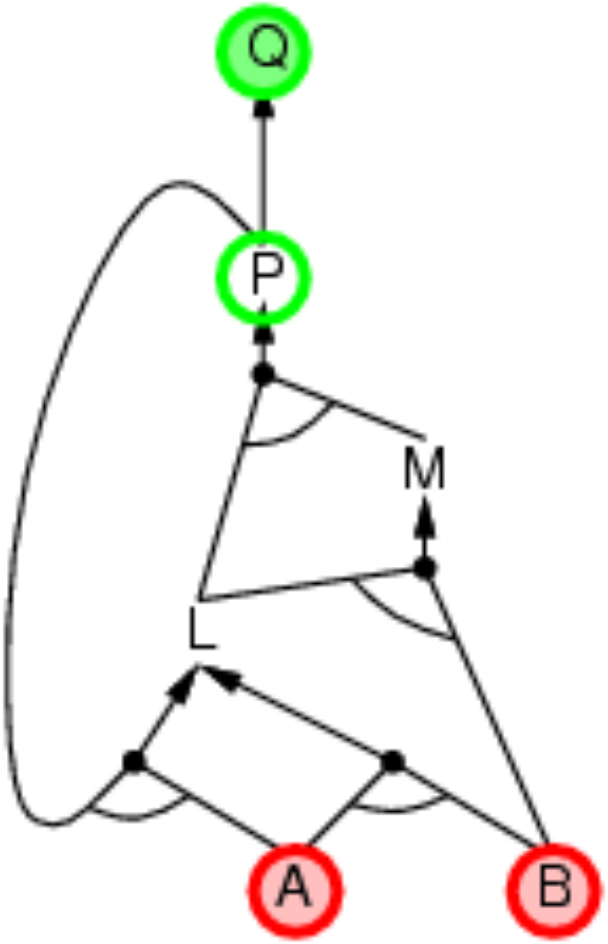
Avoid repeated work: check if new subgoal

1. has already been proved true, or
2. has already failed

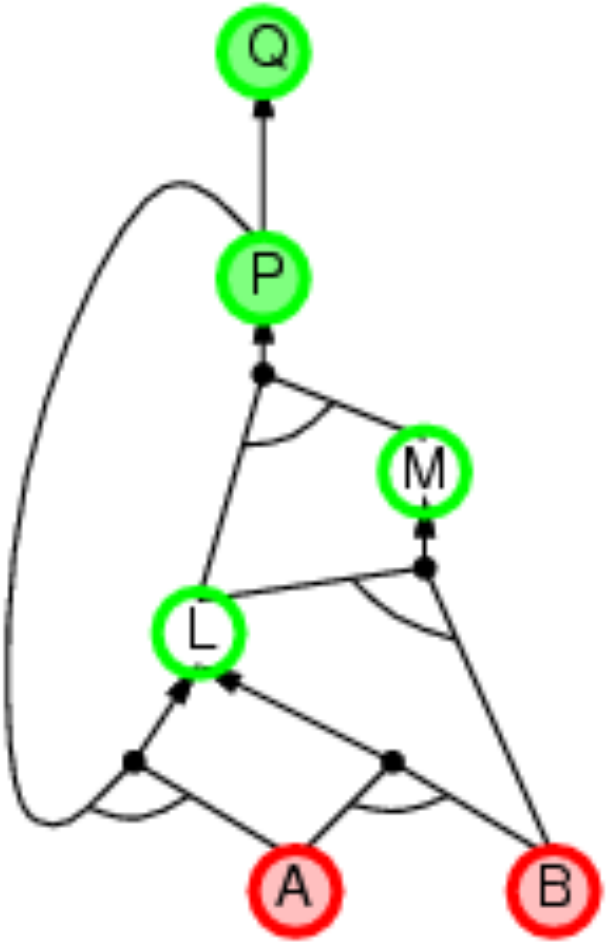
Backward chaining example



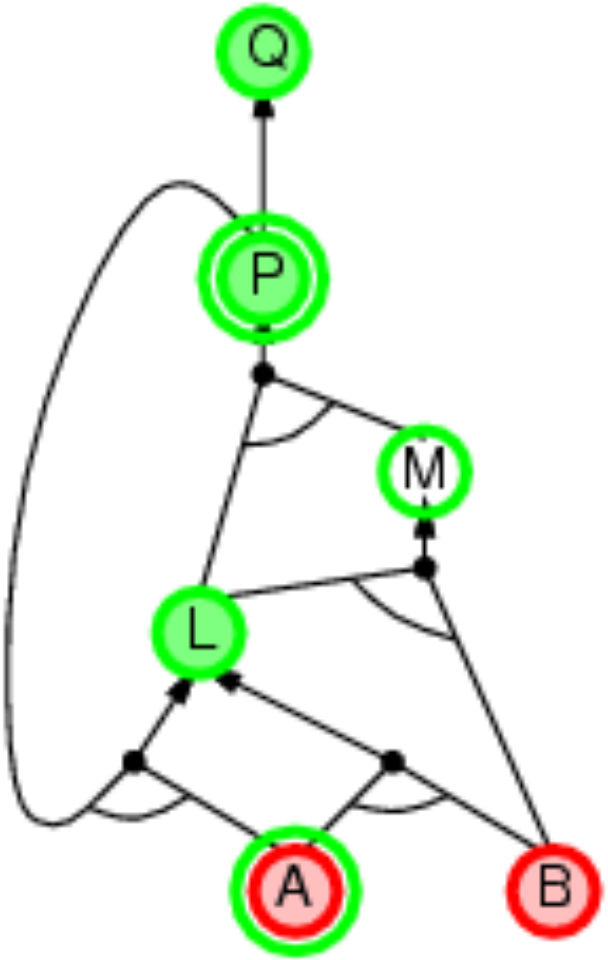
Backward chaining example



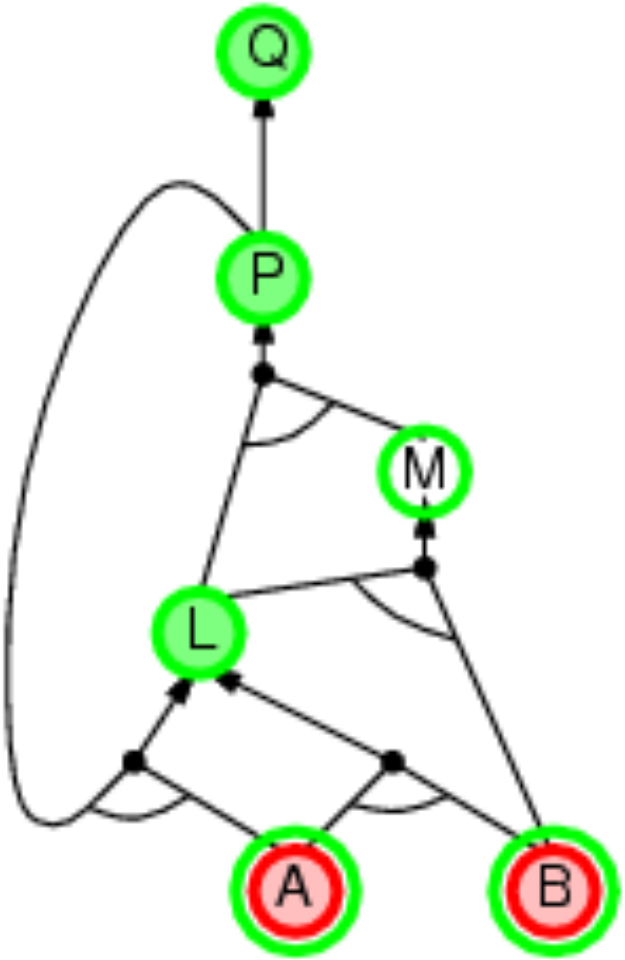
Backward chaining example



Backward chaining example



Backward chaining example



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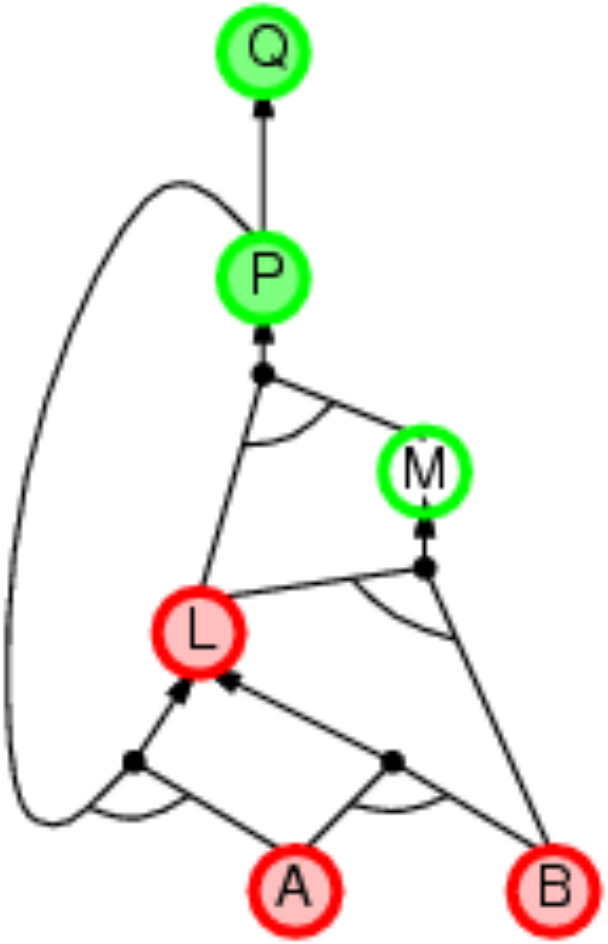
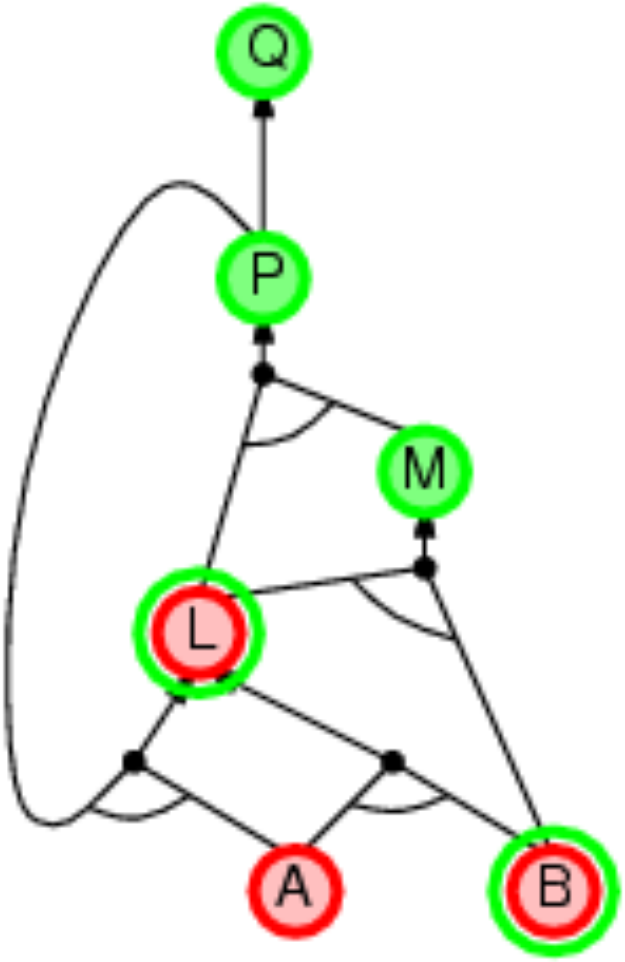
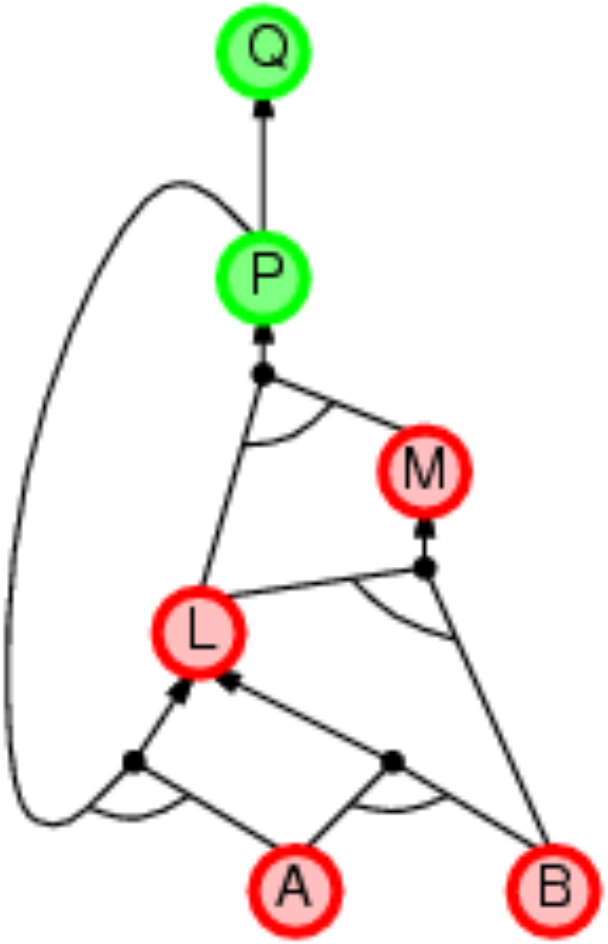


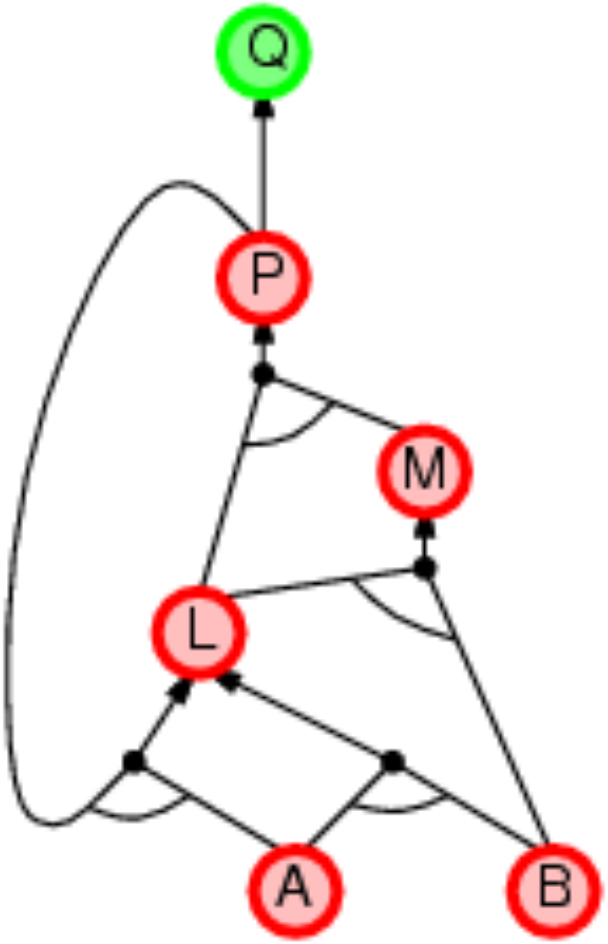
Backward chaining example



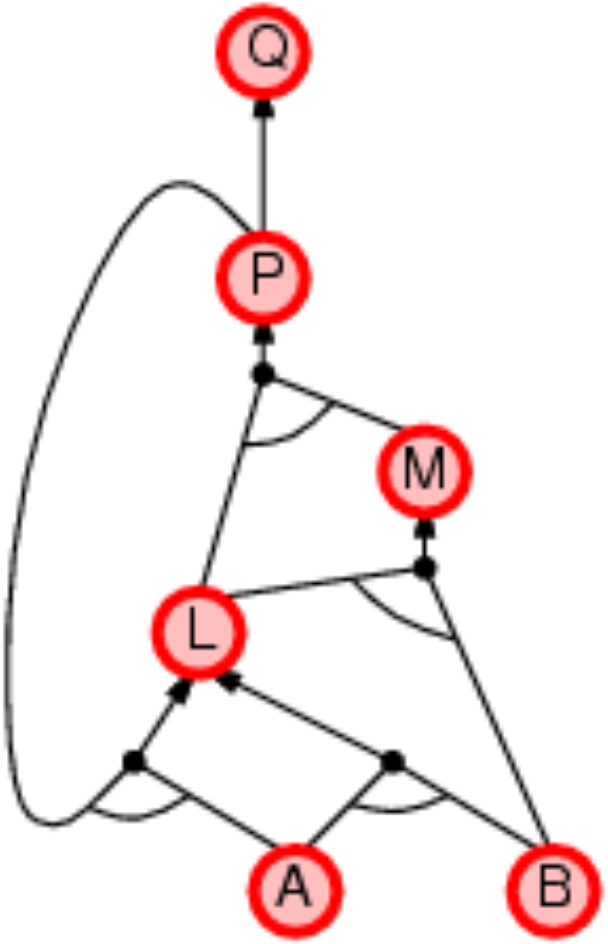
Backward chaining example



Backward chaining example



Backward chaining example



Forward vs. backward chaining

- FC is data-driven, automatic, unconscious processing,
  - e.g., object recognition, routine decisions
- May do lots of work that is irrelevant to the goal
- BC is goal-driven, appropriate for problem-solving,
  - e.g., Where are my keys?
- Complexity of BC can be much less than linear in size of KB

Efficient semantic propositional inference

Two families of efficient algorithms for propositional inference (they work at *semantic* level):

Complete backtracking search algorithms

- DPLL algorithm (Davis, Putnam, Logemann, Loveland)

Incomplete local search algorithms

- `WalkSAT` algorithm

The DPLL algorithm

Determine if an input propositional logic sentence (in CNF) is satisfiable.

Improvements over truth table enumeration:

1. **Early termination**
   A clause is true if any literal is true.
   A sentence is false if any clause is false.

2. **Pure symbol** heuristic
   Pure symbol: always appears with the same "sign" in all clauses.
   e.g., In the three clauses $(A \vee \neg B)$, $(\neg B \vee \neg C)$, $(C \vee A)$, A and B are pure, C is impure.
   Make a pure symbol literal true.

3. **Unit clause heuristic**
   Unit clause: only one literal in the clause
   The only literal in a unit clause must be true.

The DPLL algorithm

function DPLL-SATISFIABLE?(s) **returns** *true* or *false*

inputs: s, a sentence in propositional logic

clauses $\leftarrow$ the set of clauses in the CNF representation of s
symbols $\leftarrow$ a list of the proposition symbols in s
return DPLL(*clauses*, *symbols*, [])

---

function DPLL(*clauses*, *symbols*, *model*) **returns** *true* or *false*

if every clause in *clauses* is true in *model* **then return** *true*
if some clause in *clauses* is false in *model* **then return** *false*
P, *value* $\leftarrow$ FIND-PURE-SYMBOL(*symbols*, *clauses*, *model*)
if P is non-null **then return** DPLL(*clauses*, *symbols*–P, $[P = value | model]$)
P, *value* $\leftarrow$ FIND-UNIT-CLAUSE(*clauses*, *model*)
if P is non-null **then return** DPLL(*clauses*, *symbols*–P, $[P = value | model]$)
$P \leftarrow$ FIRST(*symbols*); *rest* $\leftarrow$ REST(*symbols*)
return DPLL(*clauses*, *rest*, $[P = true | model]$) **or**
DPLL(*clauses*, *rest*, $[P = false | model]$)

Esercizio DPLL-satisfiable?

(svolto in a lezione)

clauses =

$\neg P \vee \neg Q$

$\neg L \vee \neg D$

$D \vee P$

L

$\neg L \vee F \vee R$

$\neg F \vee \neg R$

symbols = ….

model = ….

Q è puro ….

The `WalkSAT` algorithm

- Incomplete, local search algorithm
- Evaluation function: The min-conflict heuristic of minimizing the number of unsatisfied clauses
- Balance between greediness and randomness

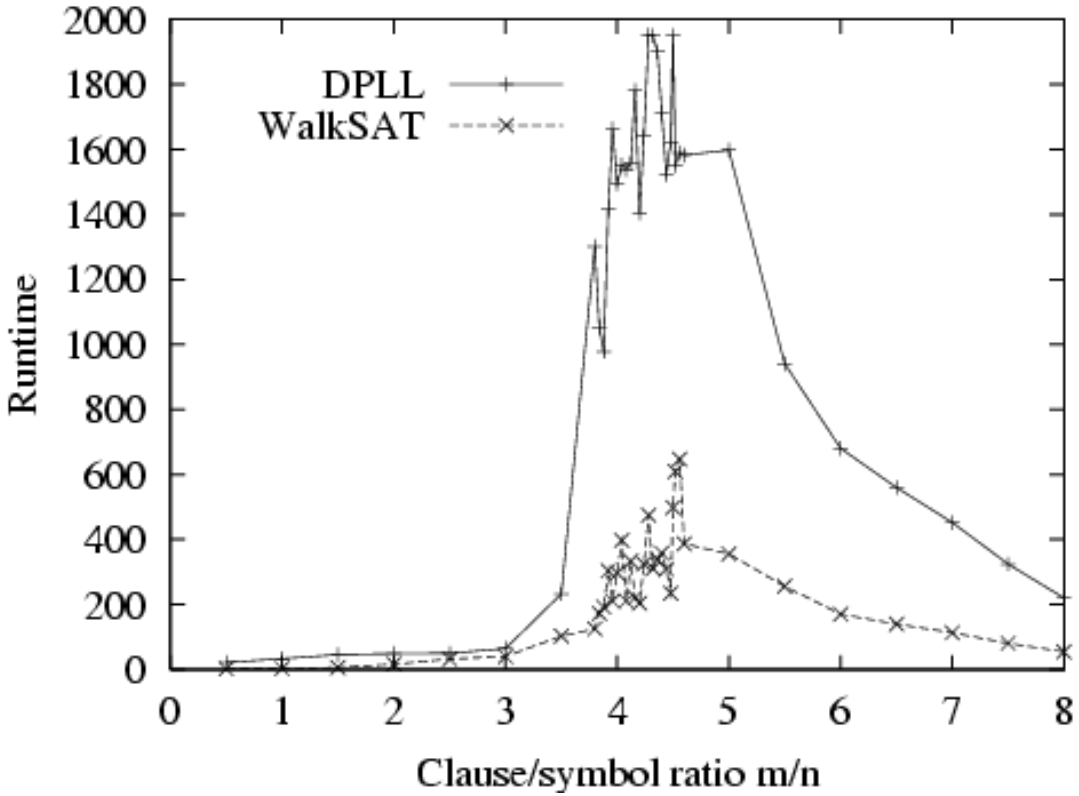
The `WalkSAT` algorithm

function WALKSAT(*clauses*, *p*, *max-flips*) **returns** a satisfying model or *failure*

inputs: *clauses*, a set of clauses in propositional logic
p, the probability of choosing to do a "random walk" move
max-flips, number of flips allowed before giving up

model ← a random assignment of *true*/*false* to the symbols in *clauses*
for $i = 1$ **to** *max-flips* **do**
  if *model* satisfies *clauses* **then return** *model*
  clause ← a randomly selected clause from *clauses* that is false in *model*
  with probability *p* flip the value in *model* of a randomly selected symbol from *clause*
  else flip whichever symbol in *clause* maximizes the number of satisfied clauses
return *failure*

Hard satisfiability problems



- Median runtime for 100 satisfiable random 3-CNF sentences, $n = 50$

Hard satisfiability problems

- Consider random 3-CNF sentences. e.g.,

  $(\neg D \vee \neg B \vee C) \wedge (B \vee \neg A \vee \neg C) \wedge (\neg C \vee \neg B \vee E) \wedge (E \vee \neg D \vee B) \wedge (B \vee E \vee \neg C)$

  m = number of clauses

  n = number of symbols

  – Hard problems seem to cluster near $m/n = 4.3$ (critical point)

Hard satisfiability problems