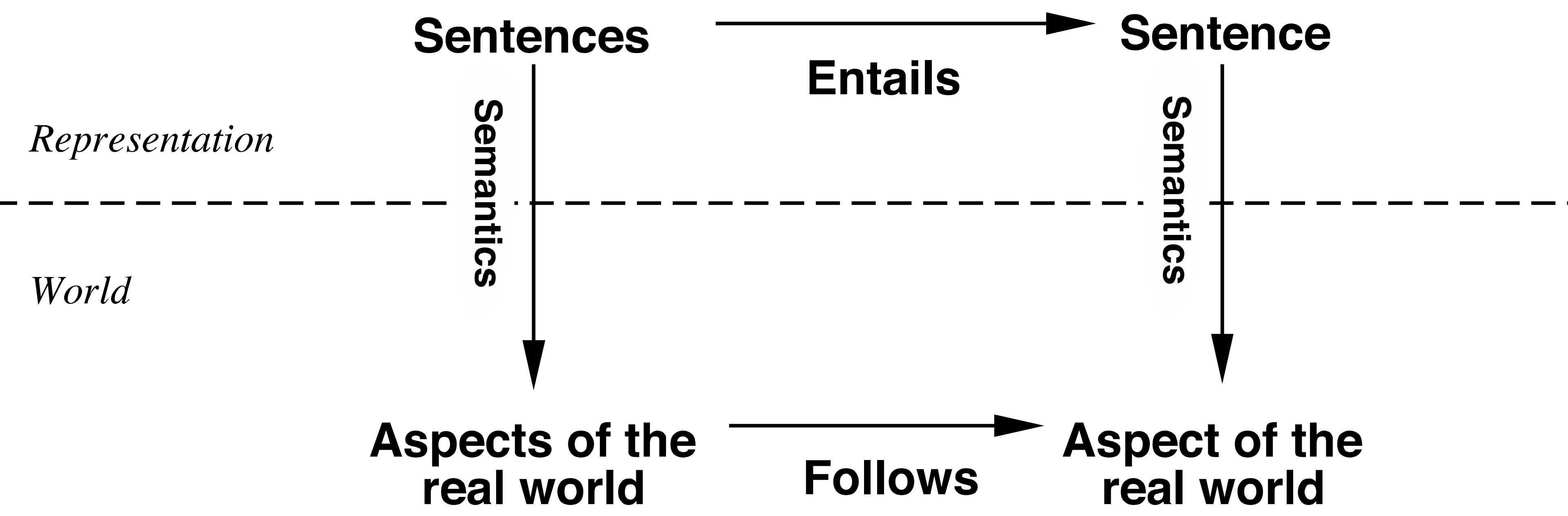
Rappresentazione e elaborazione della conoscenza: dalla sintassi (frasi della KB) alla semantica (mondo reale)



Entails: implica
Follows: conseguenza (logica)

Procedura di Inferenza/Deduzione

- "*Metodo automatico per derivare nuove frasi (formule) a partire da quelle già presenti nella base di conoscenza (KB)*"

- **Corretta** ("sound") se per ogni formula ϕ derivata da KB usando la procedura di inferenza si ha che ϕ e' una *conseguenza logica* di KB. **Completa** se vale anche il viceversa.

- **Dimostrazione** di ϕ: insieme di operazioni eseguite dalla procedura di inferenza (corretta) per derivare ϕ

- **Teoria della dimostrazione** ("proof theory"): è relativa ad un longuaggio di KR. Specifica un insieme di passi di ragionamento/inferenza che sono corretti

Alcuni Linguaggi di KR (Knowledge Representation)

- Un linguaggio di KR deve essere **conciso**, **espressivo** (il più possibile), **non ambiguo** e indipendente dal contesto, **efficace** (esiste una procedura di inferenza corretta ed implementabile)

- **Linguaggio naturale**: espressivo, ma ambiguo, non conciso

- **Linguaggio di programmazione**: preciso, strutturato, ma poco espressivo

- **Logica**: precisa, concisa, expressiva

Logica e KR

- Varie logiche, differenti assunzioni ontologiche e epistemologiche (esempi)
- Una logica è descritta da *sintassi* e *semantica*
- Vedremo Logica Proposizionale e Logica del I Ordine (Calcolo dei Predicati) in dettaglio, Logica Temporal (cenni)

Assunzioni Ontologiche/Epistemiologiche nei Linguaggi di KR

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief $\in [0, 1]$
Fuzzy logic	facts with degree of truth $\in [0, 1]$	known interval value

Figure 8.1 Formal languages and their ontological and epistemological commitments.

Sintassi logica proposizionale

$$
\begin{array}{rcl}
Sentence & \rightarrow & AtomicSentence \mid ComplexSentence \\
AtomicSentence & \rightarrow & True \mid False \mid P \mid Q \mid R \mid \ldots \\
ComplexSentence & \rightarrow & (\ Sentence\ ) \mid [\ Sentence\ ] \\
& \mid & \neg\ Sentence \\
& \mid & Sentence \land Sentence \\
& \mid & Sentence \lor Sentence \\
& \mid & Sentence \ \Rightarrow\ Sentence \\
& \mid & Sentence \ \Leftrightarrow\ Sentence \\
\\
\text{OPERATOR PRECEDENCE} & : & \neg, \land, \lor, \Rightarrow, \Leftrightarrow
\end{array}
$$

Figure 7.7 A BNF (Backus–Naur Form) grammar of sentences in propositional logic, along with operator precedences, from highest to lowest.

Semantica operatori logici

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

Figure 7.8 Truth tables for the five logical connectives. To use the table to compute, for example, the value of $P \vee Q$ when P is true and Q is false, first look on the left for the row where P is *true* and Q is *false* (the third row). Then look in that row under the $P \vee Q$ column to see the result: *true*.

Semantica della Logica

- Ciò che la formula asserisce sul mondo = **significato della frase**
- Dipende da una particolare **interpretazione**: Una formula è **vera rispetto ad una interpretazione** se ciò che rappresenta è vero nel mondo.

  Es: “*il Papa è in Francia*” ... → *“la chiave è nel frigorifero”*

- **Principio di composizionalità**: il significato di una formula è una funzione del significato delle sue parti (si per la logica!)
- **Formula valida (tautologia):** vera per qualsiasi interpretazione (“*piove **o** non piove*”. Altri esempi meno ovvi dopo
- **Formula soddisfacibile**: esiste una interpretazione per cui la formula è vera (“*piove **e** fa-caldo*”), **insoddifacibile** (“*piove **e** non piove*”) altrimenti
- Ogni formula tautologica è soddisfacibile (no vicerersa)

Modelli (logica proposizionale)

- Un **Modello** per una (formula ben formata) F.B.F ϕ è una intepretazione (associazione) dei simboli proposizionali di ϕ in valori di verità (V,F) per i quali ϕ è vero.

- Modello per ϕ: "mondo" in cui ϕ è vero rispetto ad una certa interpretazione

- Esempio, modelli di "P $\Rightarrow$ not Q": [F,F], [F,V], [V,F]

- Servono per definire il concetto (**semantico**) di **conseguenza logica**: <u>Se l'insieme di tutti i modelli di una KB di F.B.F. sono anche modelli di ϕ, allora ϕ è una conseguenza logica di KB</u>

KB = {"piove", "piove $\Rightarrow$ bagnato"} , ϕ = "bagnato"

Validity and Unsatisfiability

Validità

A sentence is **valid** if it is true in **all** interpretatioms,

Ad es. *True*, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the **Deduction Theorem**:

$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

Insoddisfacibilità

A sentence is **unsatisfiable** if it is true in **no** interpretation

e.g., $A \wedge \neg A$

Logical implication (entailment) is connected to unsatisfiability via the following: $\boldsymbol{KB \models \alpha}$ $((KB \Rightarrow \alpha)$ is valid)

if and only if $(KB \wedge \neg\alpha)$ is unsatisfiable

Metodi semantici (model checking)

- Non usano procedure di inferenza, ma modelli (dimostriamo la conseguenza logica).

- Metodo basato su **tabelle di verità** (esempi)

KB = {A, not B, (A or B $\Rightarrow$ C)} ***KB*** $\models$ ***C ?***

KB = {A, not B, (A or B $\Rightarrow$ C)} ***KB*** $\models$ ***(B*** $\Rightarrow$ ***C) ?***

- Complessità temporale e spaziale *esponenziale*.....

$KB=\{A, \text{not } B, (A \text{ or } B \Rightarrow C)\}$ $\boldsymbol{KB \models C}$?

$KB=\{A, \text{not } B, (A \text{ or } B \Rightarrow C)\}$ $\boldsymbol{KB} \models B \Rightarrow C$?

A	**B**	**C**	**not B**	**A or B ⇒ C**	**B ⇒ C**
V	V	V	F	V	V
V	V	F	F	F	F
V	F	V	V	V	V
V	F	F	V	F	V
F	V	V	F	V	V
F	V	F	F	F	F
F	F	V	V	V	V
F	F	F	V	V	V

1 MODELLO DI KB: A=V, B=F, C=V

Altro Metodo Semantico

Φ *è una conseguenza logica di una KB consistente (soddisfacibile) se e solo se KB' = KB* $\wedge$ *NOT* Φ *è insoddifacibile (non ha modelli).*

- Formulo il problema come **CSP Booleano** (**SAT**) !! Se trovo un modello per KB', allora è soddisfacibile e Φ non è una conseguenza logica di KB (altrimenti lo è)
- Complessità esponenziale nel caso generale, polinomiale per 2SAT e per *formule di Horn*

Dimostrazioni e Inferenza (ragionamento deduttivo)

- **Dimostrazione (derivazione) logica** di una formula **f**: sequenza di applicazioni di ***regole di inferenza*** a partire dalle F.B.F. nella KB fino a generare il **teorema** **f**
- **Procedura di inferenza**: procedura per costruire dimostrazioni logiche attraverso l'individuazione di appropriate *sequenze di applicazioni di regole di inferenza*
- **Regole di inferenza**: *Modus ponens, and-elimination, and-introduction, or-introduction, double-negation, unit resolution, resolution*
- Richiedono formule in *formato normalizzato*
- Esempi di dimostrazione

Resolution

Conjunctive Normal Form (CNF)
= conjunction of disjunctions of literals (= conjunction of clauses)

Example: $(A \vee \neg B)$, $(B \vee \neg C \vee \neg D)$

- Resolution inference rule (for CNF):

$$\frac{\ell_1 \vee \ldots \vee \ell_k, \qquad m_1 \vee \ldots \vee m_n}{\ell_1 \vee \ldots \vee \ell_{i-1} \vee \ell_{i+1} \vee \ldots \vee \ell_k \vee m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n}$$

where ℓ_i and m_j are *complementary* literals.

Resolution is sound and complete for propositional logic

Resolution

Implicative version of the rule

$$\frac{\neg(l_i \vee \ldots \vee l_{i-1} \vee l_{i+1} \vee \ldots \vee l_k) \Rightarrow l_i \qquad \neg m_j \Rightarrow (m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n)}{\neg(l_i \vee \ldots \vee l_{i-1} \vee l_{i+1} \vee \ldots \vee l_k) \Rightarrow (m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n)}$$

Conversion to CNF

$B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)$.
   $(B_{1,1} \Rightarrow (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1}) \Rightarrow B_{1,1})$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \lor \beta$.

   $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg(P_{1,2} \lor P_{2,1}) \lor B_{1,1})$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:

   $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land ((\neg P_{1,2} \land \neg P_{2,1}) \lor B_{1,1})$

4. Apply distributivity law ($\land$ over $\lor$) and flatten:

   $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg P_{1,2} \lor B_{1,1}) \land (\neg P_{2,1} \lor B_{1,1})$

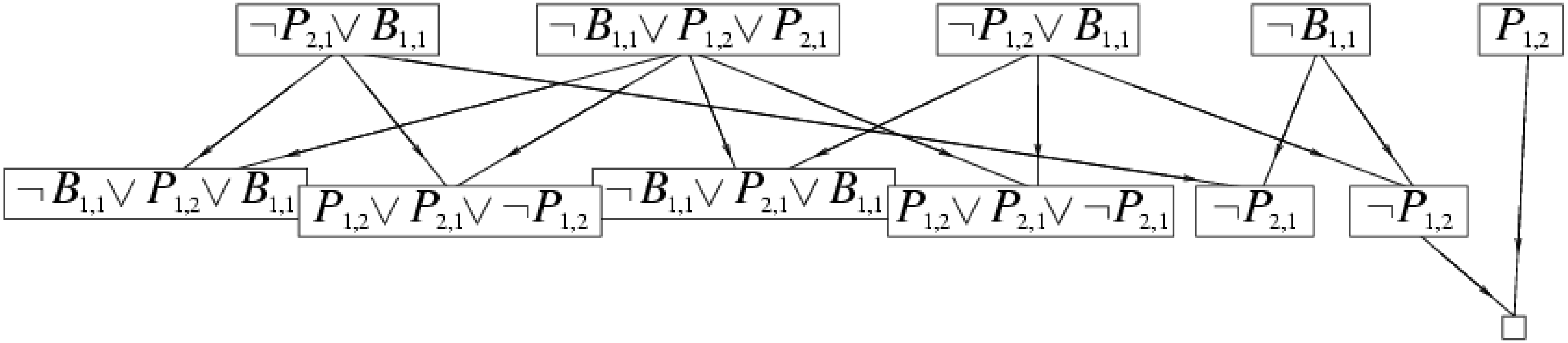
Some Logical Equivalences

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee \\
\neg(\neg \alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg \beta \Rightarrow \neg \alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg \alpha \vee \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg \alpha \vee \neg \beta) \quad \text{De Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg \alpha \wedge \neg \beta) \quad \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Figure 7.11 Standard logical equivalences. The symbols α, β, and γ stand for arbitrary sentences of propositional logic.

Resolution example

- $KB = (B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})) \land \neg B_{1,1}$
- $\alpha = \neg P_{1,2}$
-



Completezza della risoluzione

- *Una procedura di inferenza che applica la regola di risoluzione con una* ***opportuna strategia*** *è* ***completa*** *per dimostrare, attraverso il* ***metodo di refutazione, qualsiasi formula*** *della logica proposizionale*

- Esempio: Se KB = {A} non posso derivare "A or B" con la risoluzione, ma posso dimostrare "A or B" attraverso il metodo di refutazione….

- **Strategia di risoluzione =** come scelgo le 2 clausole da risolvere

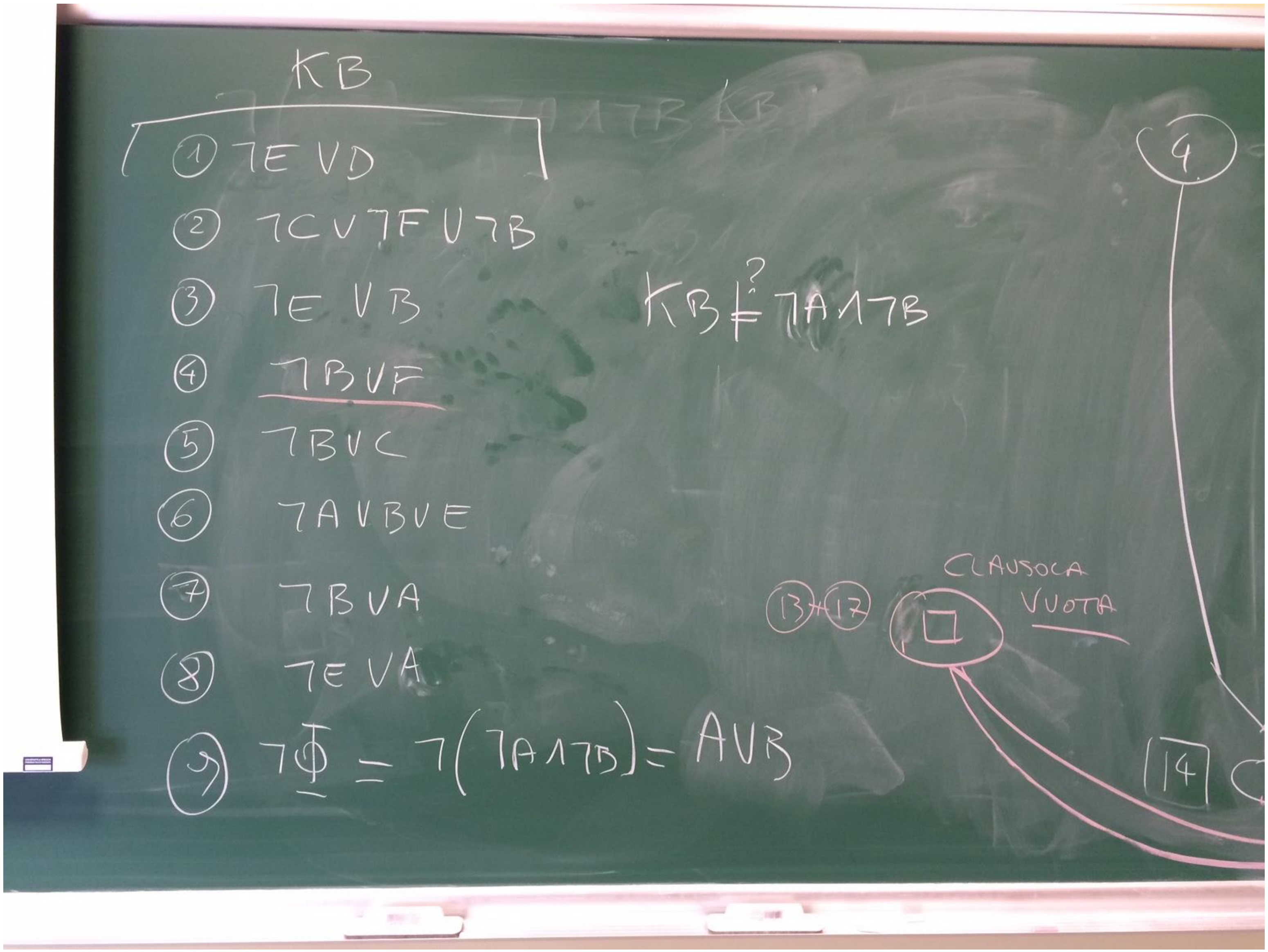
- La KB congiunta con la formula negata devono essere in **formato CNF** (congiunzione/insieme di clausole)

Resolution algorithm

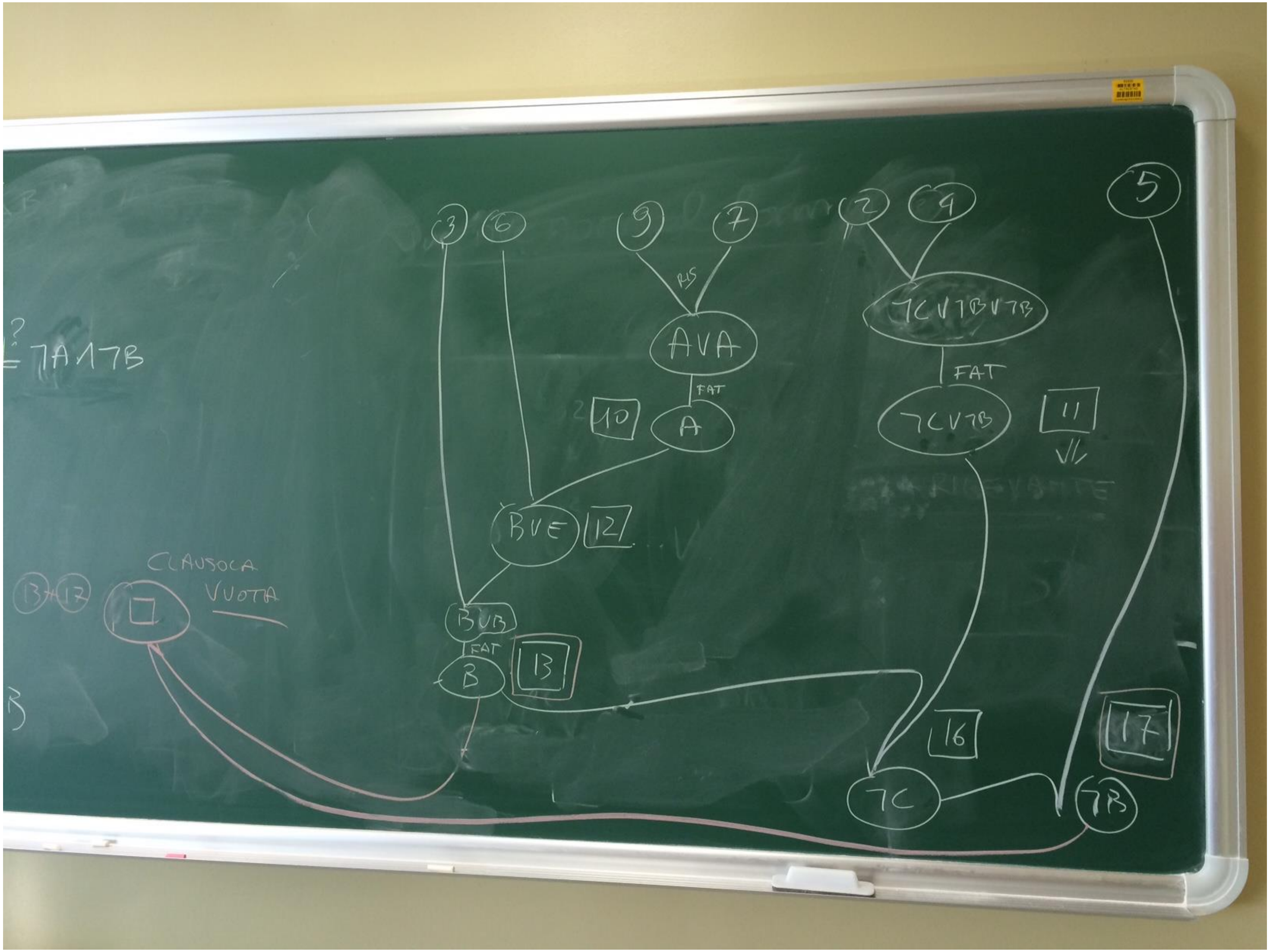
- Proof by contradiction, i.e., show $KB \wedge \neg \alpha$ unsatisfiable

```
function PL-RESOLUTION(KB, α) returns true or false
    clauses ← the set of clauses in the CNF representation of KB ∧ ¬α
    new ← { }
    loop do
        for each C_i, C_j in clauses do
            resolvents ← PL-RESOLVE(C_i, C_j)
            if resolvents contains the empty clause then return true
            new ← new ∪ resolvents
        if new ⊆ clauses then return false
        clauses ← clauses ∪ new
```

Resolution example (2)



Solution example 2 (refutation graph)



CNF, Horn, Clausole Definite

$$
\begin{aligned}
CNFSentence &\rightarrow Clause_1 \wedge \cdots \wedge Clause_n \\
Clause &\rightarrow Literal_1 \vee \cdots \vee Literal_m \\
Literal &\rightarrow Symbol \mid \neg Symbol \\
Symbol &\rightarrow P \mid Q \mid R \mid \ldots \\
HornClauseForm &\rightarrow DefiniteClauseForm \mid GoalClauseForm \\
DefiniteClauseForm &\rightarrow (Symbol_1 \wedge \cdots \wedge Symbol_l) \Rightarrow Symbol \\
GoalClauseForm &\rightarrow (Symbol_1 \wedge \cdots \wedge Symbol_l) \Rightarrow False
\end{aligned}
$$

Figure 7.14 A grammar for conjunctive normal form, Horn clauses, and definite clauses. A clause such as $A \wedge B \Rightarrow C$ is still a definite clause when it is written as $\neg A \vee \neg B \vee C$, but only the former is considered the canonical form for definite clauses. One more class is the *k*-CNF sentence, which is a CNF sentence where each clause has at most *k* literals.

Forward and backward chaining

- Horn Form (restricted)

  KB = conjunction of Horn clauses (DEFINITE CLAUSES)

  - Horn clause =
    - proposition symbol; or
    - (conjunction of symbols) $\Rightarrow$ symbol
  - E.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$

- Modus Ponens (for Horn Form): complete for Horn KBs

$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \ldots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

- Can be used with forward chaining or backward chaining.
- These algorithms are very natural and run in **linear** time!

Esercizi

$\neg P \vee Q$

$\neg L \vee \neg M \vee P$

$\neg B \vee \neg L \vee M$

$\neg A \vee \neg P \vee L$

$\neg A \vee \neg B \vee L$

A

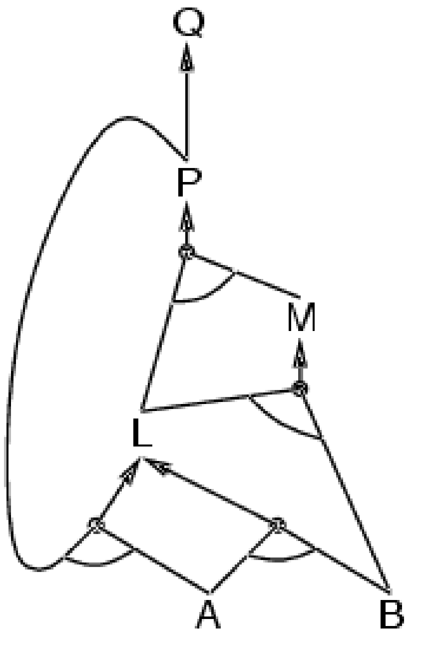
B

1. Sono clausole di Horn? Sono definite?
2. Convertite in forma implicative (come vuole modus ponens)
3. Costruire l'ipergrafo AND-OR che rappresenta KB
4. Dimostrare se $KB \models Q$ usando
   - risoluzione con strategie: **ampiezza**, clausola **unitaria** positiva, **lineare**
   - forward chaining (dopo trasformazione in clausole definite implicative)

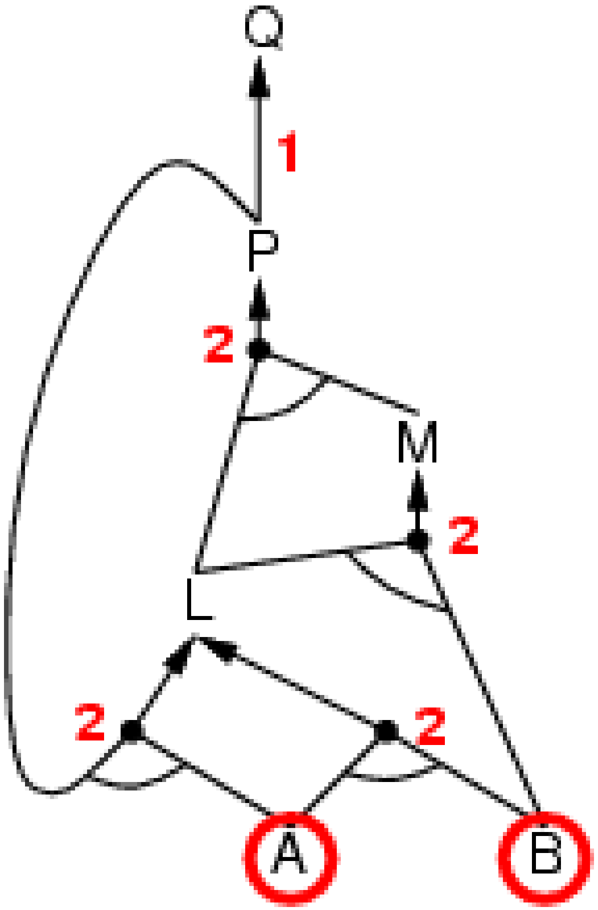
Forward chaining

- Idea: fire any rule whose premises are satisfied in the *KB*,
  - add its conclusion to the *KB*, until query is found

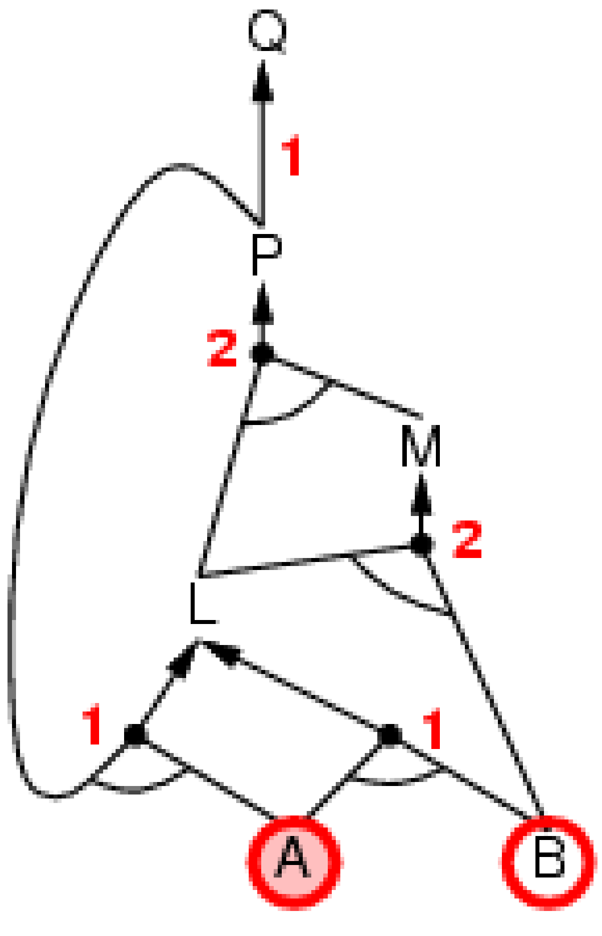
$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



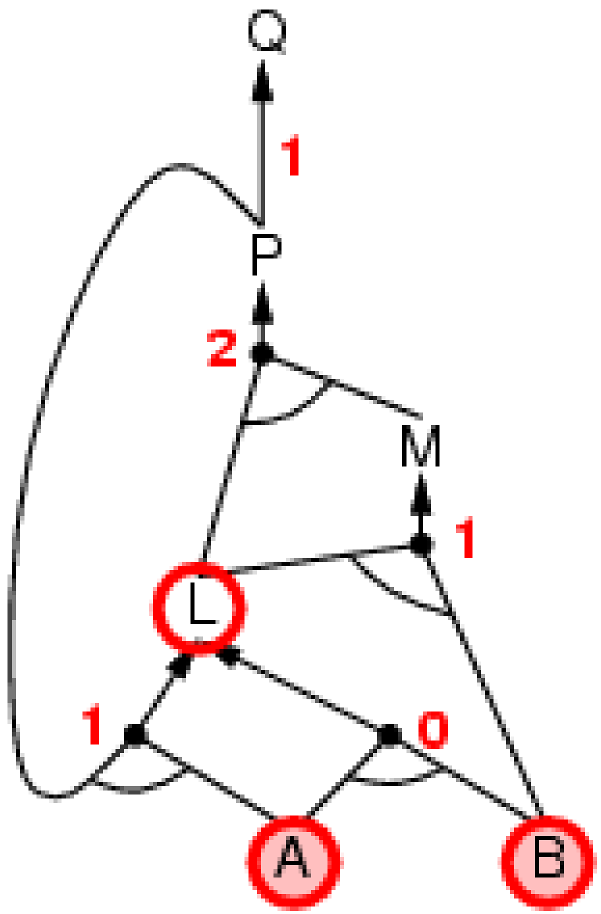
Forward chaining example



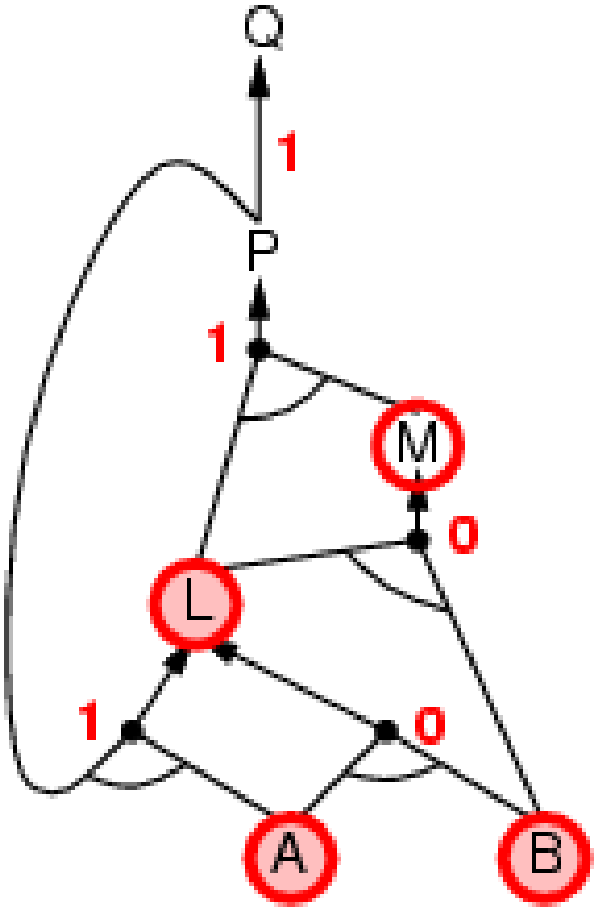
Forward chaining example



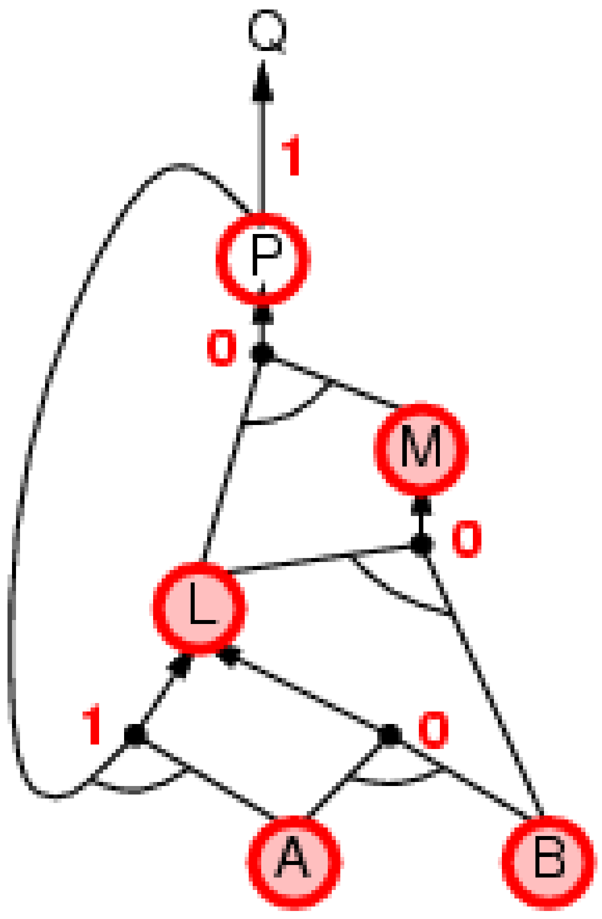
Forward chaining example



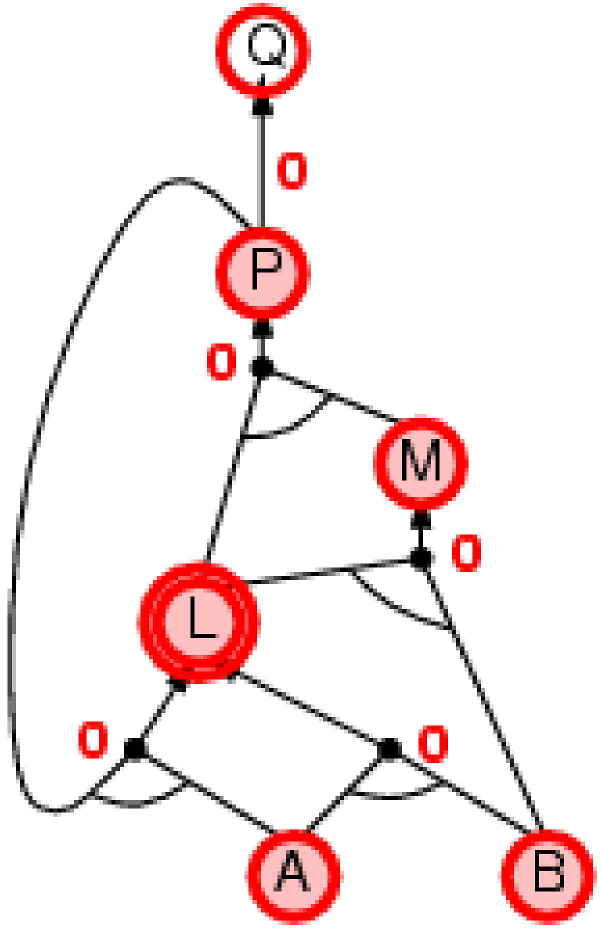
Forward chaining example



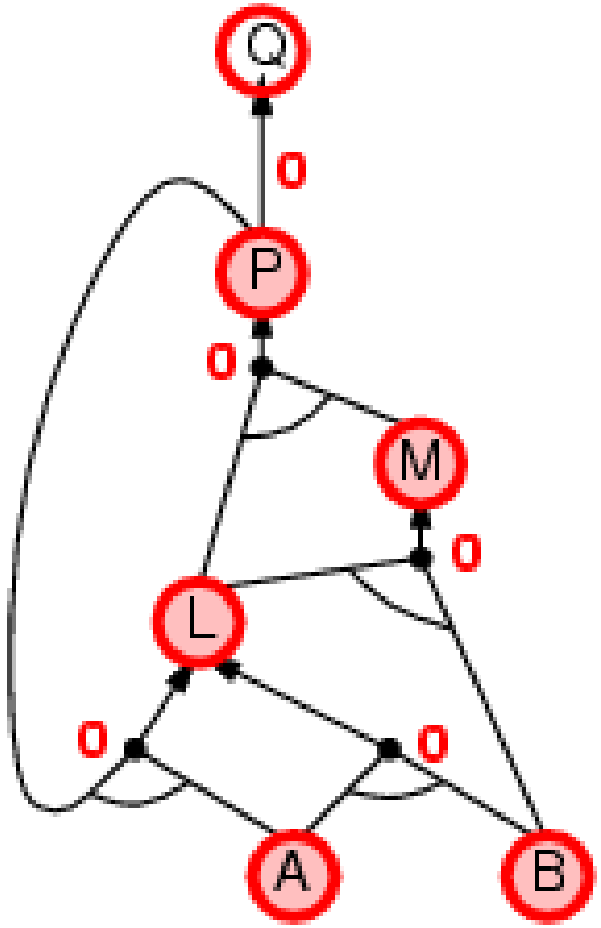
Forward chaining example



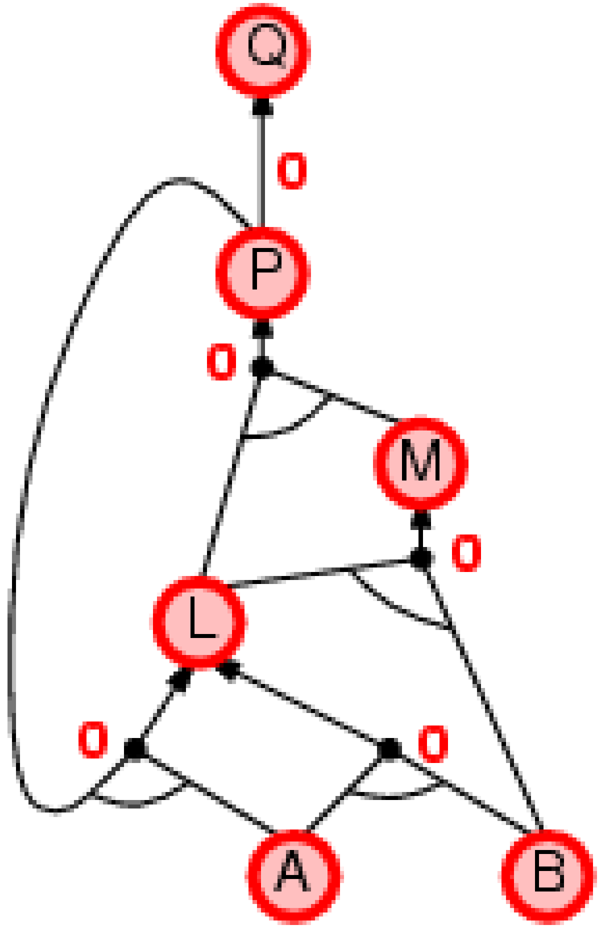
Forward chaining example



Forward chaining example



Forward chaining example



Forward chaining algorithm

```
function PL-FC-ENTAILS?(KB, q) returns true or false
  local variables: count, a table, indexed by clause, initially the number of premises
                   inferred, a table, indexed by symbol, each entry initially false
                   agenda, a list of symbols, initially the symbols known to be true

   while agenda is not empty do
       p ← POP(agenda)
       unless inferred[p] do
           inferred[p] ← true
           for each Horn clause c in whose premise p appears do
               decrement count[c]
               if count[c] = 0 then do
                   if HEAD[c] = q then return true
                   PUSH(HEAD[c], agenda)
   return false
```

- Forward chaining is sound and complete for Horn KB

Proof of completeness

FC derives every atomic sentence that is entailed by *KB*

1. FC reaches a fixed point where no new atomic sentences are derived
2. Consider the final state as a model *m*, assigning true/false to symbols
3. Every clause in the original *KB* is true in *m*

   (if $a_1 \land \ldots \land a_k \Rightarrow b$ is *false* in *m* FC has not reached fixed point)
4. Hence *m* is a model of *KB*
5. If $KB \models q$, *q* is true in every model of *KB*, including *m*

   Perciò è impossibile che FC **non** derivi q; se così fosse, allora per costruzione di **m** esisterebbe un modello (m) di KB in cui *q* = *false* e, per definizione di $\models$, **m** dovrebbe essere anche modello di *q*, che è impossibile

Backward Chaining (BC)

Idea: work backwards from the query q:

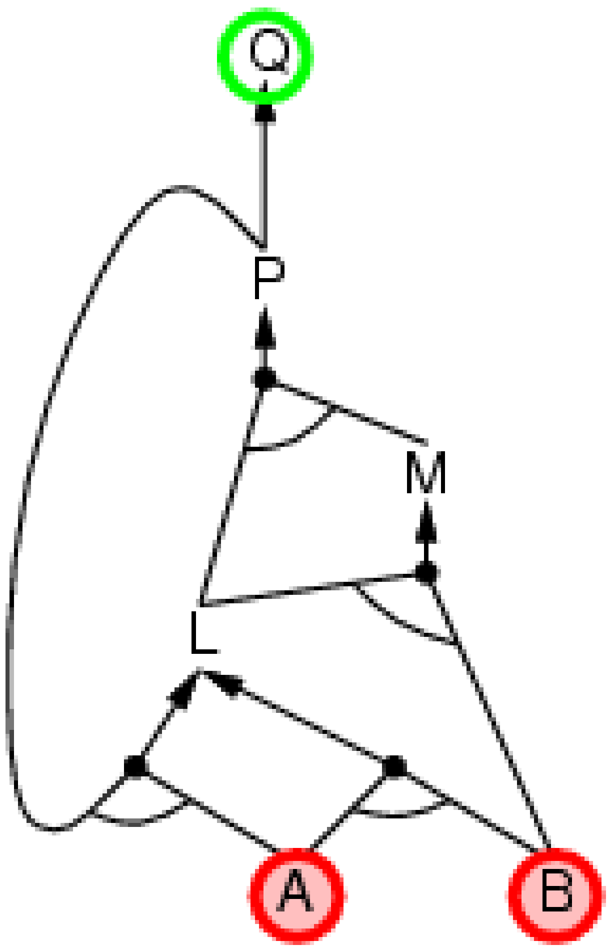
- to prove q by BC,
  - check if q is known already, or
  - prove by BC all premises of some rule concluding q

Avoid loops: check if new subgoal is already on the goal stack

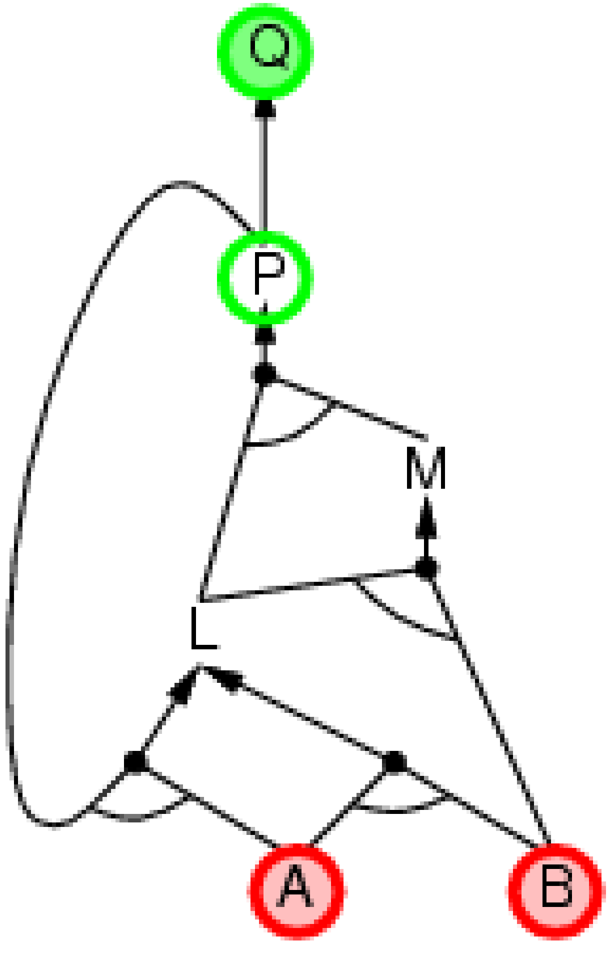
Avoid repeated work: check if new subgoal

1. has already been proved true, or
2. has already failed

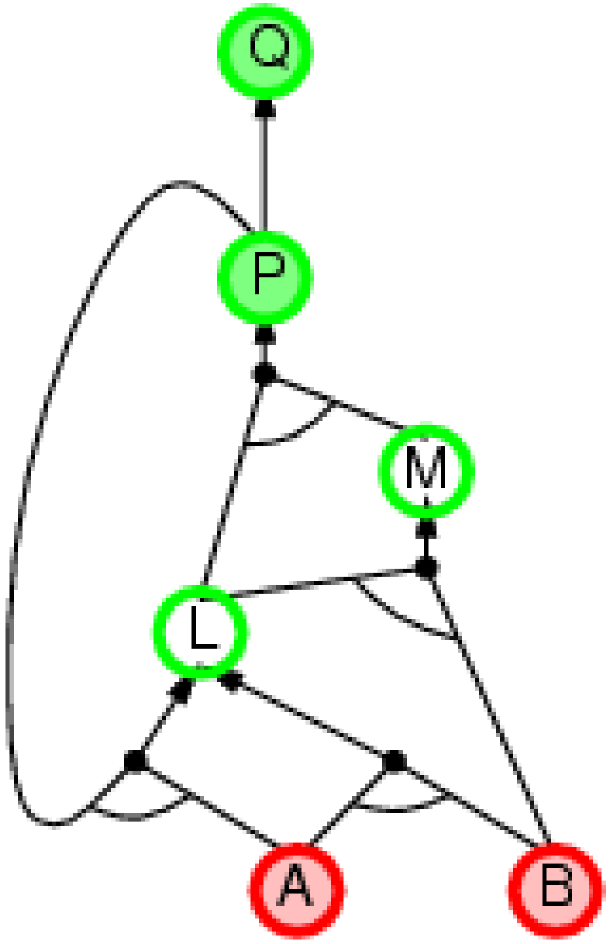
Backward chaining example



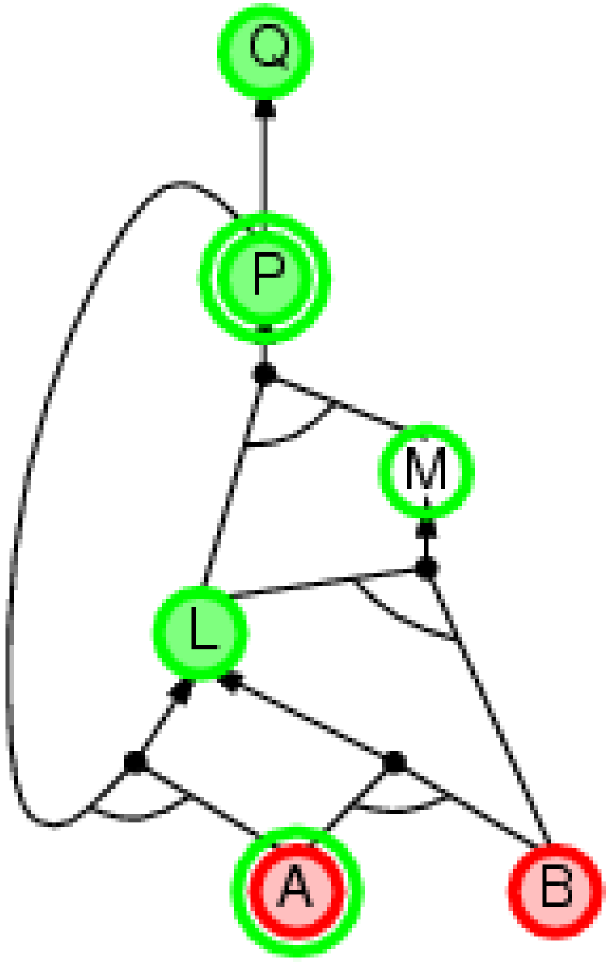
Backward chaining example



Backward chaining example



Backward chaining example



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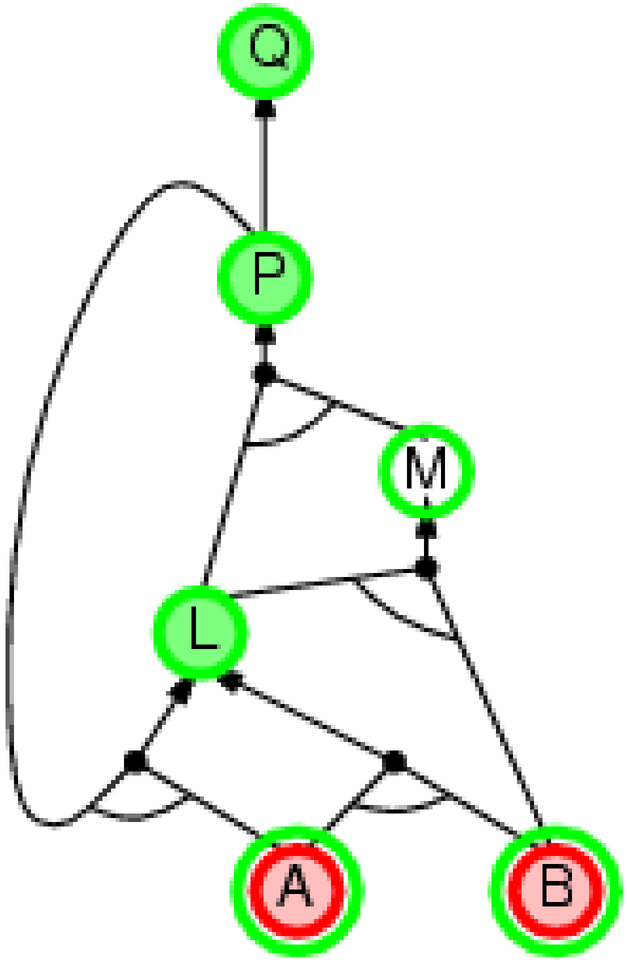
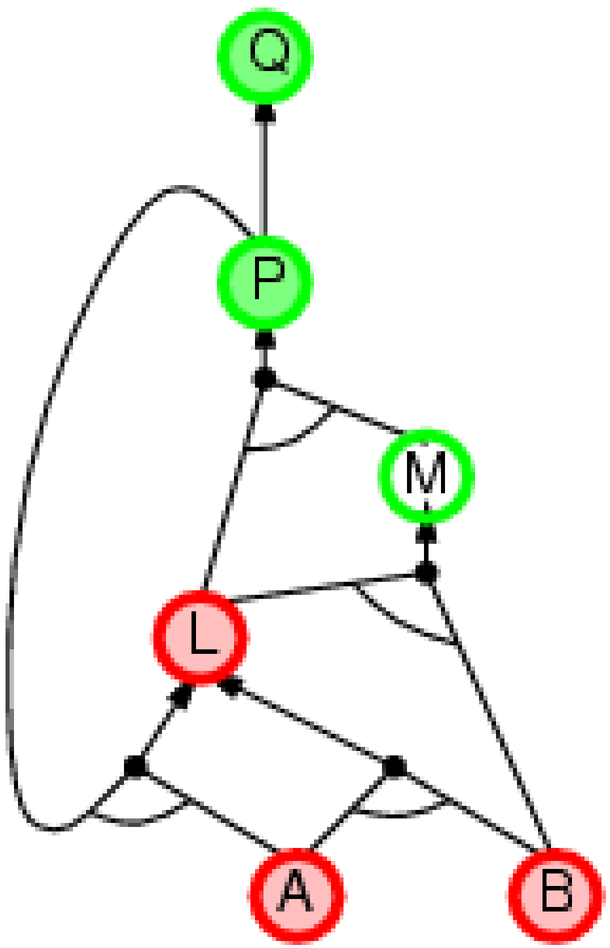
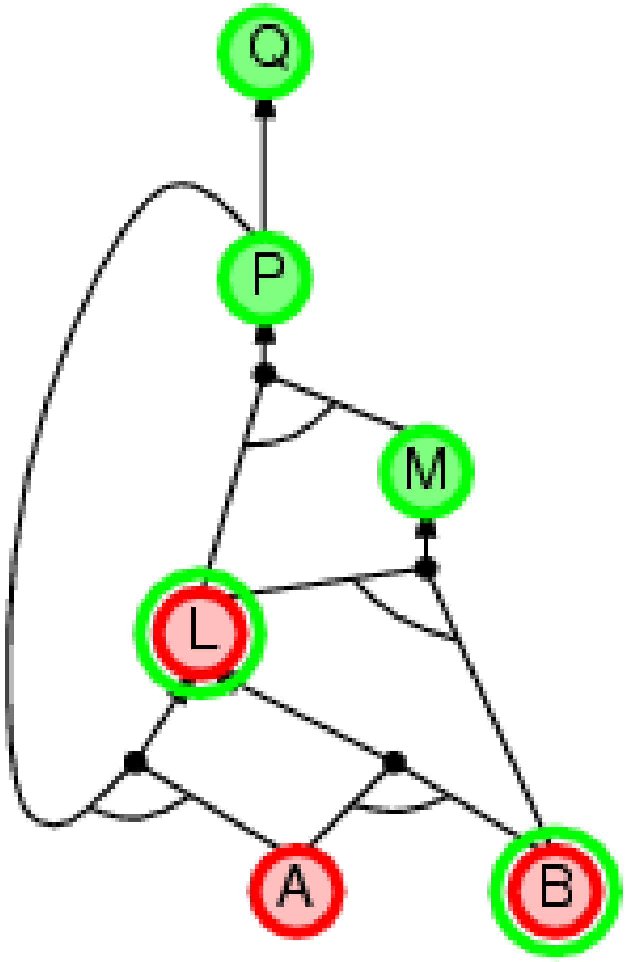


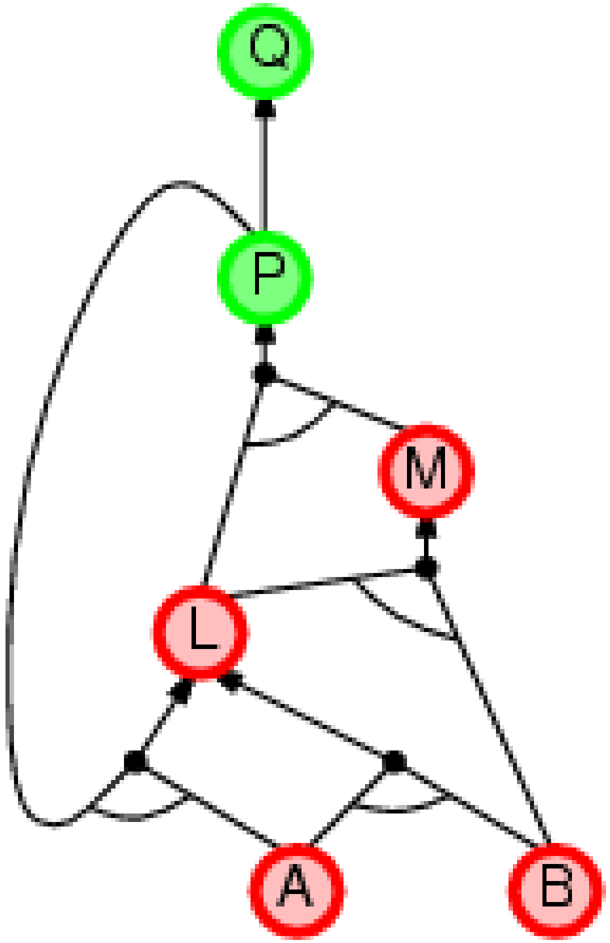
Backward chaining example



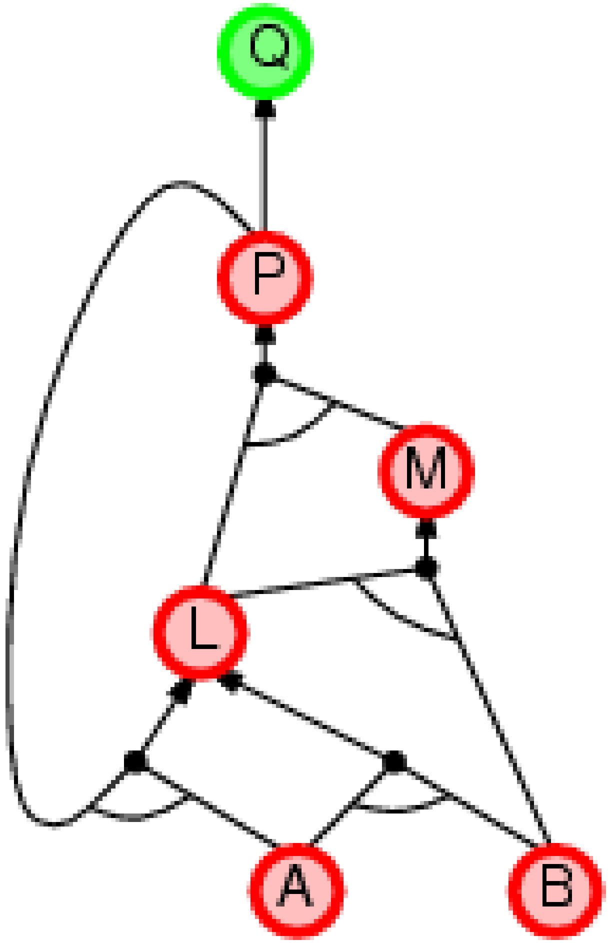
Backward chaining example



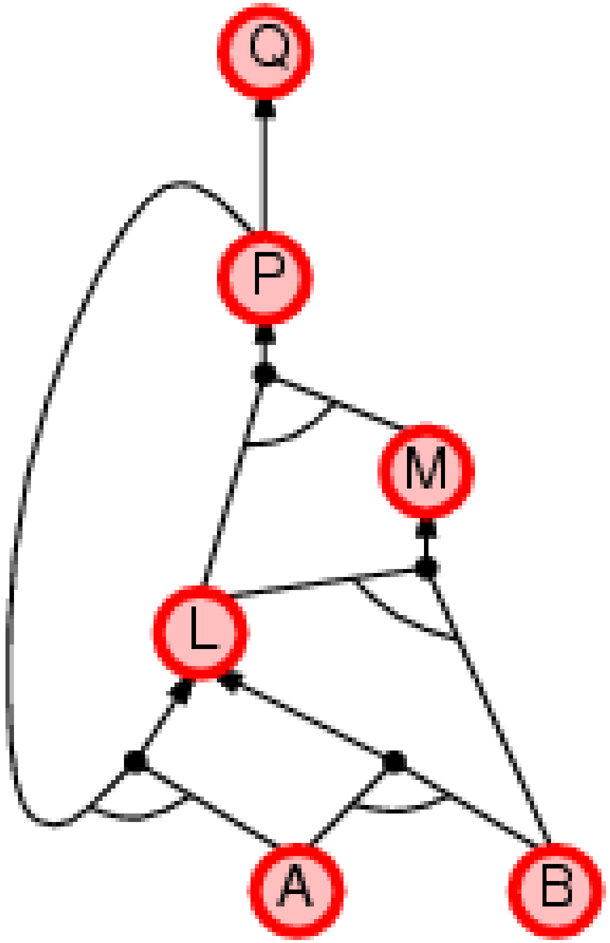
Backward chaining example



Backward chaining example



Backward chaining example



Forward vs. backward chaining

- FC is data-driven, automatic, unconscious processing,
  - e.g., object recognition, routine decisions
- May do lots of work that is irrelevant to the goal
- BC is goal-driven, appropriate for problem-solving,
  - e.g., Where are my keys?
- Complexity of BC can be much less than linear in size of KB

Efficient semantic propositional inference

Two families of efficient algorithms for propositional inference (they work at *semantic* level):

Complete backtracking search algorithms

- DPLL algorithm (Davis, Putnam, Logemann, Loveland)

Incomplete local search algorithms

- `WalkSAT` algorithm

The DPLL algorithm

Determine if an input propositional logic sentence (in CNF) is satisfiable.

Improvements over truth table enumeration:

1. **Early termination**
   A clause is true if any literal is true.
   A sentence (conjunction of clauses) is false if any clause is false.

2. **Pure symbol** heuristic
   Pure symbol: always appears with the same "sign" in all clauses.
   e.g., In the three clauses $(A \vee \neg B)$, $(\neg B \vee \neg C)$, $(C \vee A)$, A and B are pure, C is impure.
   Make a pure symbol literal true.

3. **Unit clause heuristic**
   Unit clause: only one literal in the clause
   The only literal in a unit clause must be true.

The DPLL algorithm

function DPLL-SATISFIABLE?(s) **returns** *true* or *false*
  inputs: s, a sentence in propositional logic

  clauses ← the set of clauses in the CNF representation of s
  symbols ← a list of the proposition symbols in s
  return DPLL(*clauses*, *symbols*, [ ])

---

function DPLL(*clauses*, *symbols*, *model*) **returns** *true* or *false*

  if every clause in *clauses* is true in *model* **then return** *true*
  if some clause in *clauses* is false in *model* **then return** *false*
  P, *value* ← FIND-PURE-SYMBOL(*symbols*, *clauses*, *model*)
  if *P* is non-null **then return** DPLL(*clauses*, *symbols*–*P*, [*P* = *value*|*model*])
  P, *value* ← FIND-UNIT-CLAUSE(*clauses*, *model*)
  if *P* is non-null **then return** DPLL(*clauses*, *symbols*–*P*, [*P* = *value*|*model*])
  P ← FIRST(*symbols*); *rest* ← REST(*symbols*)
  return DPLL(*clauses*, *rest*, [*P* = *true*|*model*]) **or**
      DPLL(*clauses*, *rest*, [*P* = *false*|*model*])

Esercizio DPLL-satisfiable?

(svolto in aula a lezione)

clauses = symbols = …. model = ….

$\neg P \lor \neg Q$
$\neg L \lor \neg D$
$D \lor P$
L
$\neg L \lor F \lor R$
$\neg F \lor \neg R$

Q è puro ….

`WalkSAT` algorithm

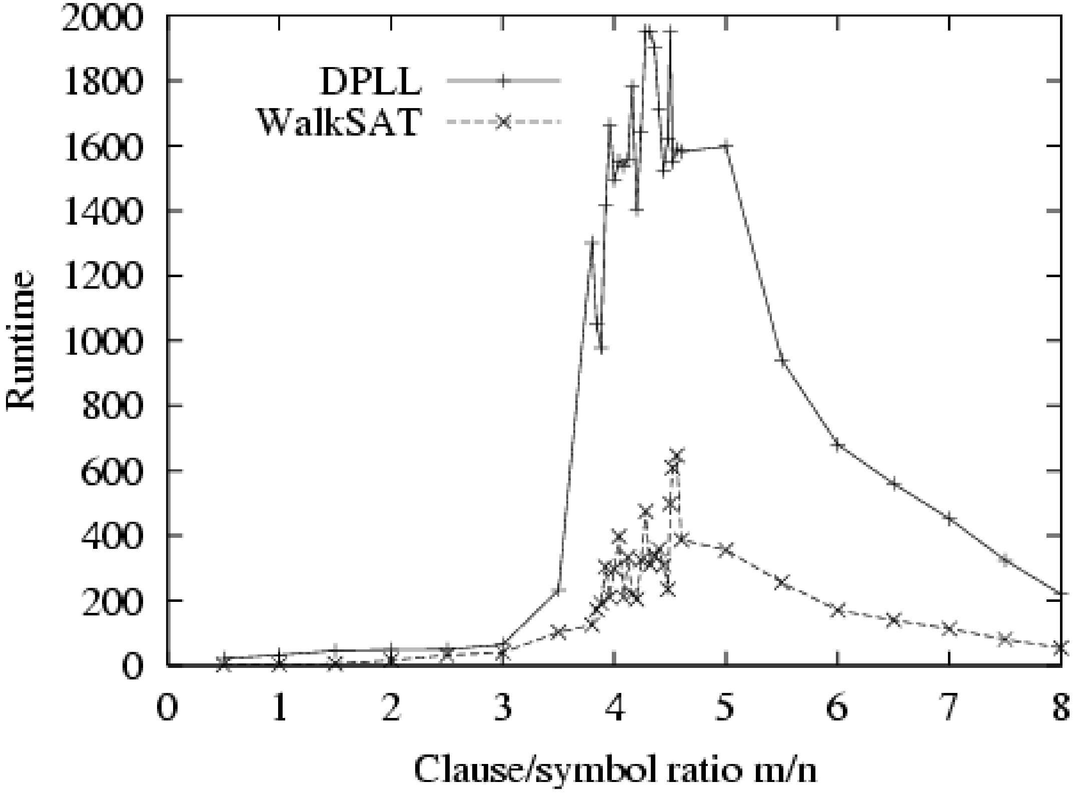
- Incomplete, local search algorithm
- Evaluation function: The min-conflict heuristic of minimizing the number of unsatisfied clauses
- Balance between greediness and randomness

The `WalkSAT` algorithm

```
function WALKSAT(clauses, p, max-flips) returns a satisfying model or failure
   inputs: clauses, a set of clauses in propositional logic
           p, the probability of choosing to do a "random walk" move
           max-flips, number of flips allowed before giving up

   model ← a random assignment of true/false to the symbols in clauses
   for i = 1 to max-flips do
      if model satisfies clauses then return model
      clause ← a randomly selected clause from clauses that is false in model
      with probability p flip the value in model of a randomly selected symbol
            from clause
      else flip whichever symbol in clause maximizes the number of satisfied clauses
   return failure
```

Hard satisfiability problems



- Median runtime for 100 satisfiable random 3-CNF sentences, $n = 50$

Hard satisfiability problems

- Consider random 3-CNF sentences. e.g.,

  $(\neg D \lor \neg B \lor C) \land (B \lor \neg A \lor \neg C) \land (\neg C \lor \neg B \lor E) \land (E \lor \neg D \lor B) \land (B \lor E \lor \neg C)$

  m = number of clauses

  n = number of symbols

  – Hard problems seem to cluster near $m/n = 4.3$ (critical point)

Hard satisfiability problems