Ragionamento Spaziale

Variabili: **regioni** dello spazio

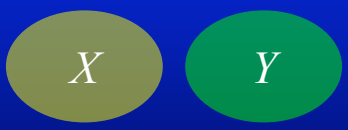
Relazioni binarie per esprimere:

- La **posizione** relativa (destra, sinistra, nord, sud, ecc.)
- La **topologia** (disconnesse, attaccate, sovrapposte, ecc.)
- La **dimensione** relativa (qualitativa o quantitativa)
- La **distanza** relativa (qualitativa o quantitaiva)
- ….

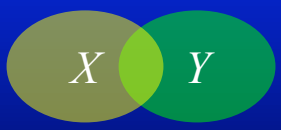
Disgiunzioni di relazioni per esprimere conoscenza indefinita

Ogni tipo di relazione ha associato un calcolo, che puo' essere combinato con altri.

RCC-5: 5 Basic Topological Relations Between Regions

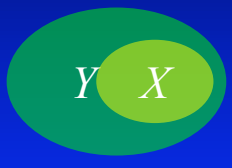


X DC *Y*

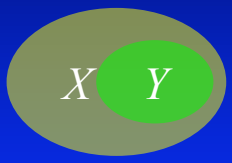


X PO *Y*

X EQ *Y*

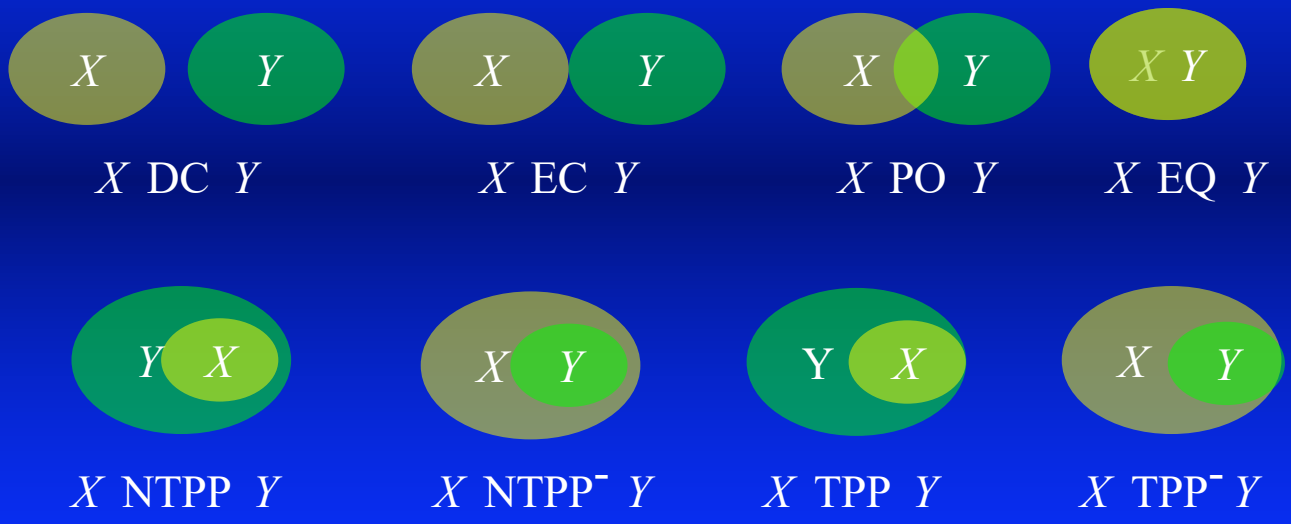


X PP *Y*



X PP⁻ *Y*

RCC-8: 8 Basic Topological Relations Between Regions



RCC-5: Tabella Composizione Relazioni di Base

	DC	PO	PP	PP-	EQ
DC	*	DC,PO,PP	DC,PO,PP	DC	DC
PO	DC,PO,PP	*	PP,PO	DC,PO,PP	PO
PP	DC	DC,PO,PP	PP	*	PP
PP-	DC,PO,PP-	PO,PP-	PO,PP,PP-	PP-	PP-
EQ	DC	PO	PP	PP-	EQ

RCC-5/8 Relations: Semantics, Syntax & Reasoning

- **Region variables** denote *regular, closed subsets* of a *topological space (for instance R^2 or R^3)*
- **Indefinite knowledge** expressed by *union of relations, w*ritten as *sets* of base relations, e.g.

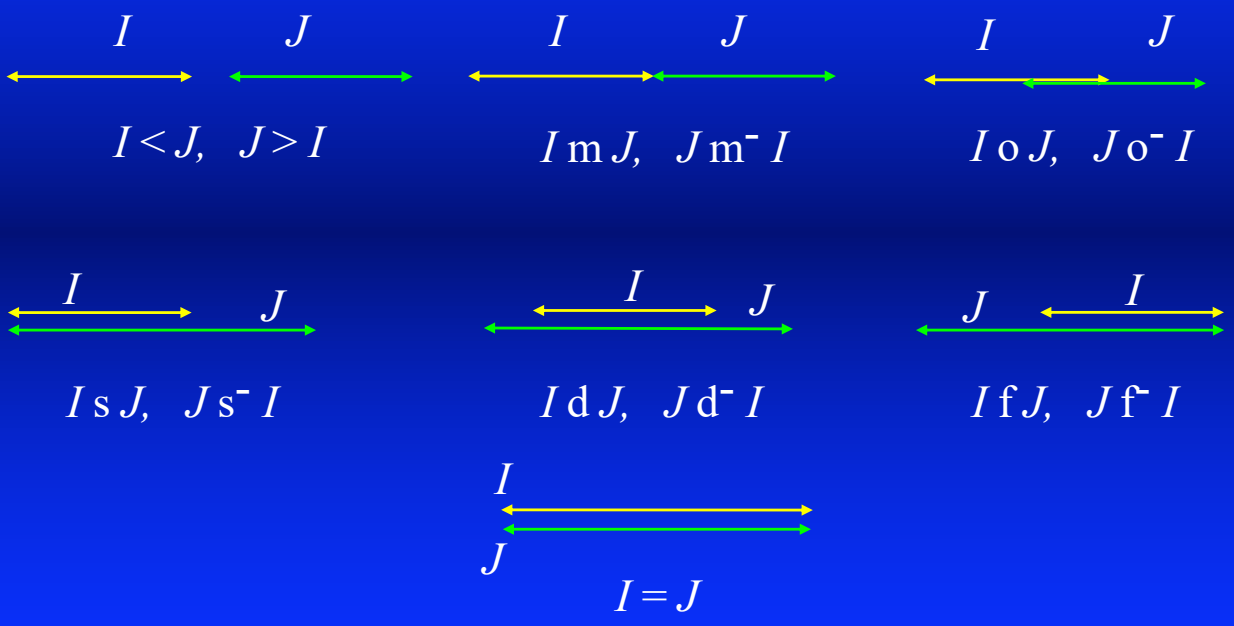
  X {PO, EC} *Y*

  → **2^8** different relations for RCC-8 and **2^5 for RCC-5**
- Main **inference task:** Given a set of topological formulae (a spatial CSP), is this set *satisfiable?*

RCC-8: Computational Properties

- Satisfiability is **NP-complete (also for RCC-5!)**
- **3 maximal tractable fragments** containing the *universal* relation and all *base* relations: satisfiability is decidable by the *path-consistency algorithm* in $O(n^3)$ time [Renz & Nebel '99, Renz '99]
- These fragments can be extended with *qualitative relations (<, = ,≤ ,≠) between the size of the regions*: Satisfiability is still decidable in $O(n^3)$ time by the *Bipath-consistency algorithm* [Gerevini & Renz '98]

Allen 's Basic Interval Relations



Interval Relations: Semantics, Syntax & Reasoning

- **Interval variables** denote pairs of rational numbers
- **Indefinite knowledge** is expressed by unions of relations (written as sets of base relations):

  I {s,d,f} J

  → **8192** different relations
- Main **inference task:** Given a set of temporal formulae (a temporal CSP), is this set *satisfiable*?

Refinements & Scenarios For Qualitative Constraint Formulae

- Given a set of constraint formulae Θ (i.e., a CSP), we say Θ ' is a **refinement** of Θ, if all formulae in Θ ' are stronger (fewer base relations)

$$(I \{s,d,f\} J), (J \{<, >\} K) \rightarrow (I \{s\} J), (J \{>\} K)$$

$$(I \{PO,DC\} J), (J \{EC,PO\} K) \rightarrow (I \{PO\} J), (J \{PO\} K)$$

- A satisfiable CSP Θ is called **scenario** if there is a constraint with only *one base relation* between **each pair** of variables

$$(I \{s\} J), (J \{>\} K), (I \{>\} K)$$

$$(I \{PO\} J), (J \{PO\} K), (I \{DC\} K)$$

A Spatio-Temporal Constraint Calculus (Temporalizzazione di RCC-8 con IA)

- RCC-8 topological relationships between **regions** hold over a **temporal interval**
- Intervals are related using Allen 's relations
- **STCC CSP**: set of annotated RCC-8 constraint formulae, and Allen's constraints on their intervals

I: $(X \{EC,DC\} Y)$, $(Y \{TPP\} Z)$

J: $(X \{PO\} Y)$, $(Y \{DC\} Z)$

$I \{<, m\} J$

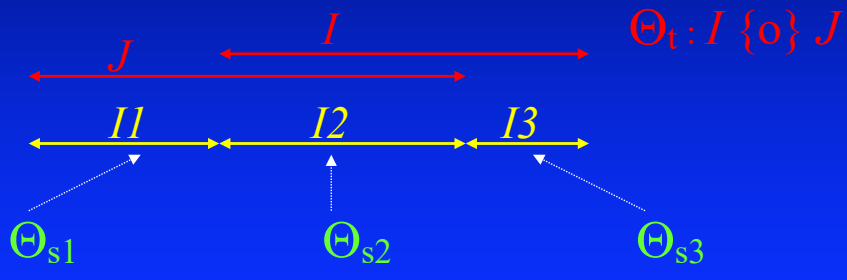
STCC CSPs: Semantics

- **Interpretation of a STCC CSP (Q,T,α):**
  - **Q**: rational numbers
  - T: topological space
  - α: maps interval symbols to pairs of numbers from **Q**, and region symbols and rational numbers to non-empty, regular, closed subsets of T
- **Model (or Solution) of a STCC CSP:**

  Interpretation such that: all temporal constraints are satisfied, and each spatial formula annotated with *I* is satisfied at every point *between the endpoints of the interval* associated with *I*

Induced Scenario of a STCC CSP Θ

- Derive a temporal scenario Θ_t for the temporal CSP
- From Θ_t derive a total order of interval endpoints and the *induced* set of sub-intervals I1, I2, ..., Ik
- Derive a spatial scenario Θ_{Si} for each sub-interval Ii (Θ_{s2} satisfies *both* the formulae annotated *I* and *J*)



Satisfiability of STCC CSPs: RISAT

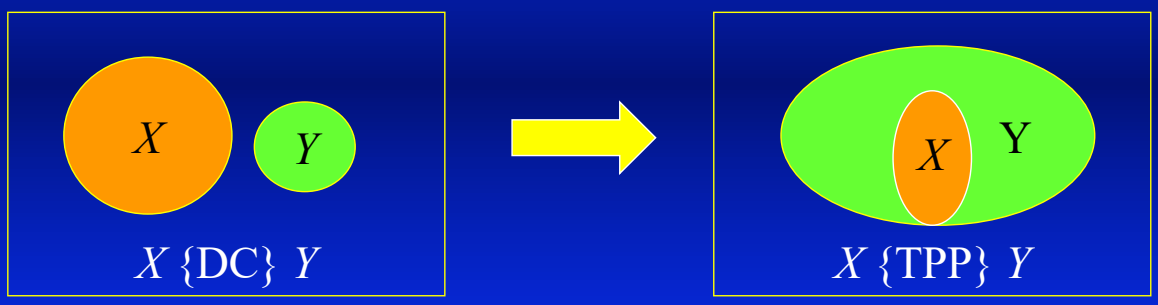
RISAT is NP-complete

Theorem: RISAT *is* NP*-hard even if the CSP contains only basic relations and the two universal relations.*

Theorem: *When the temporal relations form a scenario and the spatial relations are all elements of one tractable class,* RISAT *is polynomial.*

Spatial Change

When spatial configurations change over time, are all changes meaningful?



- *X* and *Y* change their *shape*
- *X* and *Y* change their ***size***
- The topological change is ***not continuous***

The Size Persistence Constraint

The **size** of every region persists over time (cannot be expressed in the STCC formalism)

I: **(*X* {TPP} *Y*)** ***J:*** **(*X* {EQ} *Y*)**

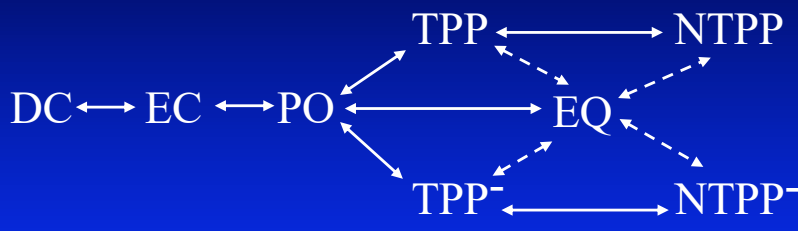
is not satisfiables regardless of the temporal relation between *I* and *J*

Is satisfiability still NP-complete ?

Theorem: RISAT *for a STCC CSP* Θ *with the size persistence constraint is* NP*-complete.*

The Continuity Constraint

- Going from one Spatial Scenario to the next, the topological relations *must change continuously* (neighborhood structure [Freksa]):



- We may also want to preserve the size of all regions (no dashed lines)

STCC with the Continuity Constraint

- Problema computazionalmente ancora aperto! Si *congettura* che decidere la soddisfacibilità sia un problema NP-completo